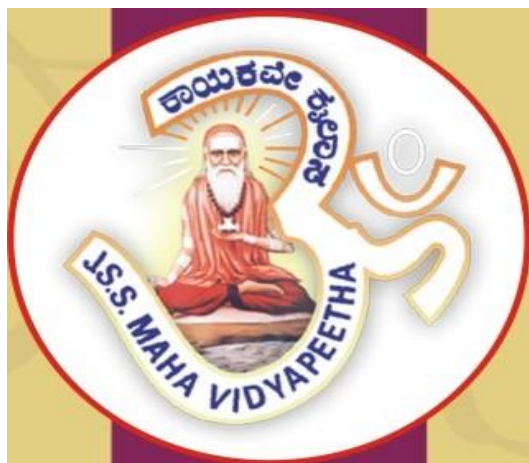


JSS COLLEGE FOR WOMEN (AUTONOMOUS)

Saraswathipuram, Mysore-9



BACHELOR OF SCIENCE GRADUATE COURSE

SYLLABUS FOR DSC AND OPEN ELECTIVES

(I to VI Semesters)

UNDER NEP

SUBJECT: PHYSICS

Inclusion of the Syllabus

Sl.No	Index
1	Proceedings of the Board of Studies
2	Composition of board of studies
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JSS College for Women (Autonomous), Saraswathipuram, Mysuru-09

PROCEEDINGS

Proceedings of the of the **Board of Studies in Physics** meeting held on 14th August 2023.

The meeting of Board of Studies in Physics was convened at 11A.M. on 14th August 2023 in the Department of Physics, JSS College for Women (Autonomous), Saraswathipuram, Mysuru-09

The following members were Present in the meeting

1. Smt.Sowmya B, Assistant Professor, JSS College for Women (Autonomous), Mysuru.
2. Prof.M S Chandrashekar, Professor, DOS in Physics, Manasagangothri, Mysuru.
3. Dr.H R Sreepad, Associate Professor, Govt. College (Autonomous), Mandya
4. Dr.M V Manjunath, Associate Professor, Maharani's Science College, Mysuru.
5. Dr.S R Kumaraswamy, Associate Professor, Maharani's Science College, Mysuru
6. Dr.Chandra, Assistant Professor, NIE College, Manathavadi Road, Mysuru
7. Sri.Umesh V, Assistant Professor, JSS College for Women (Autonomous), Mysuru.
8. Kum.Divya A, Assistant Professor, MMS & SDM Mahila Vidyalaya, Mysuru.

The following member took leave of Absence

1. Sri.E Manivannan, Project Assistant, Mani Electronics, K.T.Street, Mysuru

With subject to the UG Physics NEP Syllabus of V and VI semesters provided by the University of Mysore, all the members of the Board discussed in length and made a deep analysis on various topics and recommended valuable suggestions to frame the Theory part, Practical part and Activity part for the new **NEP Scheme** Syllabus for V and VI semesters for the Academic year 2023-2024 and onwards.

In the V and VI semester syllabus two new experiments were added to the practical part.

The Board of studies approved the syllabus which includes DSC-5 and DSC-6 major papers of 4 credits (4 hours) each and two practical papers of 2 credits (4hours) each, examination pattern, department regulations, the panel of examiners and question paper pattern.

The Board also approved the Emplotability Skills Syllabus for SEC Paper and Internship (Either External /Internal mode) for VI Semester students for 2 Credits.

This scheme should be effective from the academic year 2023-2024.

The copy of the B.Sc Physics Syllabus of V to VI Semesters under NEP Scheme is annexed herewith.

PREFACE

The syllabus proposed herein for the B.Sc., course in Physics has evolved through detailed discussions with members of Physics faculty in our college as well as many other Physicists and Physics teachers. The changes proposed have constantly kept in mind certain basic approaches in Physics education along with the dynamism resulting from autonomy in education. We are very much aware that Physics forms a major driving force for the present day developments in technology and the consequent socio-economic development of the world at large. The rapid changes in technology and vast variety of the present day technologies put enormous pressure on the practitioners to ensure that their education is extensive as well as intensive. Consequently there is a need for constant upgrading and revision of basic inputs in education at all levels, ensuring a judicious mix of the topics chosen. We have taken the advantage of impetus gained through autonomy, the consequent academic freedom and the possibility of achieving high quality at the institutional level. We are equally aware of the immense responsibility it entails in ensuring a proper match between the products of the education and the professions they might get into restrictions arising primarily from considerations of the available time, manpower and financial resources have also been emphasized during the formulation of the contents. We also believe that there is a vast scope for future improvements and a pressing need for constant up gradation and revision of the contents. We would also urge that the individual teachers feel free to add, delete or modify topics of their choice and provide the relevant feedback to help improve the content formulation.

We thank and gratefully acknowledge the help we have received from all the member of the Physics community and a special thanks to all the members of the board of Studies.

Department Regulations

1. Teaching instruction per week:

For I to VI Semester:

“Course duration: 16 weeks with 4 hours of instructions per week.”

a) Lecture classes: DSC- 4 Hrs of duration per week (One Paper).

b) Practical classes: DSC- 4 Hrs of duration per week (One Practical).

2. Examination:

a) Theory:

C1- Will be assessed for I to VI semester for 20 marks after the completion of first half of the semester by 8th week through Test/Activity/Assignment/Attendance/Seminar/Projects in group and Poster presentation (after the completion of 50% of the syllabus).

C2- Will be assessed for I to VI semester for 20 marks after the completion second half of the semester by 16th week through Test/Activity/Assignment/Attendance/Seminar/Projects in group and poster presentation (after the completion of rest 50% of the syllabus).

C3- Examinations for I to VI semester are conducted at the end of every semester for TWO hours of duration. The question paper shall be set for a maximum of 60 marks from I to VI semester.

b) Practical:

C1- Will be assessed for I to VI semester for 10 marks after the completion of first half of the semester by 8th week through experiment/continuous assessment of experimental work and record completion/procedure writing/viva/ attendance (after the completion of 50% of the experiments).

C2- Will be assessed for I to VI semester 15 marks after the completion of second half of the semester by 16th week through experiment/continuous assessment of experimental work and record completion/procedure writing/viva/ attendance (after the completion of all the experiments).

C3- Examinations for I to VI semester are conducted at the end of every semester for THREE hours of duration for a maximum of 25 marks ONE

experiment shall be conducted in each practical examination.

3. **Eligibility criteria for students :**

Only the students who have scored minimum of 40% in C1 and C2 put together are eligible to take C3 examination.

4. **Eligibility criteria for teaching faculty:**

a) **Paper setting**-the teacher with minimum of 5 years of teaching experience in the first grade college are eligible to set the question paper.

b) **Paper valuation and Practical examination** – the teacher with minimum of 3 years of experience is eligible to become an evaluator and examiner.

5. **Internship:** In VI Semester internship will be guided by the in house teacher for 2 Credits. Students can complete their internship by preparing a dissertation report (either internal work / External work modes) on a project / lecture by an external subject expert/ Field visit/industrial visit/ research lab visit/ Any other relevant mode related to Core Subject based on the need for student's interest.

Course Structure (Core and Open Electives)

Sem	Course	Paper	Title of the Paper	Instructions per week (Hrs)	Credits	Components Max. Marks			Total Marks	Exam Duration (Hrs)
					L:T:P	C1	C2	C3		
I	DSC-1	Theory	Mechanics and properties of Matter	4	4:0:0	20	20	60	100	02
		Practical	Mechanics and properties of Matter	4	0:0:2	10	15	25	50	03
	OE - 1	Theory	Energy Sources	3	3:0:0	20	20	60	100	02
	OE - 2	Theory	Climate Science	3	3:0:0	20	20	60	100	02
II	DSC-2	Theory	Electricity and Magnetism	4	4:0:0	20	20	60	100	02
		Practical	Electricity and Magnetism	4	0:0:2	10	15	25	50	03
	OE - 3	Theory	Astronomy	3	3:0:0	20	20	60	100	02
	OE - 4	Theory	Medical Physics	3	3:0:0	20	20	60	100	02
III	DSC-3	Theory	Wave Motion and Optics	4	4:0:0	20	20	60	100	02
		Practical	Wave Motion and Optics	4	0:0:2	10	15	25	50	03
	OE - 5	Theory	Optical Instruments	3	3:0:0	20	20	60	100	02
	OE - 6	Theory	Sports Science	3	3:0:0	20	20	60	100	02
IV	DSC-4	Theory	Thermal Physics and Electronics	4	4:0:0	20	20	60	100	02
		Practical	Thermal Physics and Electronics	4	0:0:2	10	15	25	50	03
	OE - 7	Theory	Nanotechnology	3	3:0:0	20	20	60	100	02
	OE - 8	Theory	Electrical Instruments	3	3:0:0	20	20	60	100	02

Sem	Course	Paper	Title of the Paper	Instructions per week (Hrs)	Credits	Components Max. Marks			Total Marks	Exam Duration (Hrs)
					L:T:P	C1	C2	C3		
V	DSC-5	Theory	Classical Mechanics and Quantum Mechanics - I	4	4:0:0	20	20	60	100	02
		Practical	Classical Mechanics and Quantum Mechanics - I	4	0:0:2	10	15	25	50	03
	DSC-6	Theory	Elements of Atomic, Molecular and Laser Physics	4	4:0:0	20	20	60	100	02
		Practical	Elements of Atomic, Molecular and Laser Physics	4	0:0:2	10	15	25	50	03
	SEC	Theory	Employability skills	3	2:0:1	25	----	25	50	01
VI	DSC-7	Theory	Elements of Condensed Matter & Nuclear Physics	4	4:0:0	20	20	60	100	02
		Practical	Elements of Condensed Matter & Nuclear Physics	4	0:0:2	10	15	25	50	03
	DSC-8	Theory	Electronic Instrumentation & Sensors	4	4:0:0	20	20	60	100	02
		Practical	Electronic Instrumentation & Sensors	4	0:0:2	10	15	25	50	03
	INTERNSHIP	Internship	Internship (Internal/External)	2	0:0:2	50	50	-----	100	-----

I Semester (Core)

Mechanics & Properties of Matter

Course Title: Mechanics & Properties of Matter	Course Credits: 4
Total Contact Hours: 52	Duration of ESA: 2 hours
Formative Assessment Marks: 40	Summative Assessment Marks: 60
Model Syllabus Authors:	Physics Expert Committee

Mechanics & Properties of Matter		Hrs
Credit : 4+2	Unit – 1	Theory : 4 hours /Week
Chapter No. 1	Frames of references: Inertial and non inertial reference frames, A frame moving with uniform velocity with respect to another inertial frame is also inertial, an accelerated frame of reference with respect to an inertial frame is non inertial, Galilean transformation equations, Lorentz transformation equations.	(13)
Chapter No. 2	Momentum and Energy: Work and energy, Conservation of momentum (linear). Conservation of energy with examples. Motion of rockets.	
Chapter No. 3	Special Theory of Relativity: Constancy of speed of light. Postulates of Special Theory of Relativity. Length contraction. Time dilation. Relativistic addition of velocities.	
Topics for self study(If any)	Self Study Units and measurements: System of units (CGS and SI), measurement of length, mass and time, dimensions of physical quantities, dimensional formulae. Minimum deviation, errors.	
	Suggested Activities	
Activity No. 1	1. i). Students can measure diameters of small balls of different size and estimate their volumes. 2. ii). Students can measure lengths of nails of different size. iii). Students can measure volume of a liquid iv). Students can measure distances and put the result both in CGS and SI units in 2, 3 and 4 significant figures. Ask them to mention the precession of the measurement. v). students can estimate standard deviations wherever possible.	

Activity No. 2	Students can try and understand conservation of energy in everyday examples. For example: i) What happens in solar conservation panels ii) Pushing an object on the table it moves iii) Moving car hits a parked car causes parked car to move. In these cases, energy is conserved. How? Understand and verify if possible.	
Unit – 2		
Chapter No. 4.	Laws of Motion: Newton's Laws of motion. Dynamics of single and a system of particles. Centre of mass.	(13)
Chapter No. 5.	Dynamics of Rigid bodies: Rotational motion about an axis, Relation between torque and angular momentum, Rotational energy. moment of inertia: $M I$ of a rectangular Lamina and solid cylinders. Flywheel, Theory of compound pendulum and determination of g .	
Chapter No. 6.	Gravitation: Law of Gravitation. Motion of a particle in a central force field (motion is in a plane, angular momentum is conserved, areal velocity is constant). Kepler's laws (statements). Satellite in a circular orbit.	
Topics for self study(If any)	Chapter 7: Geosynchronous orbits. Basic idea of global positioning system (GPS). Ref: 1-4,9,10	
	Suggested Activities	
Activity No. 3	Activity: Moment of inertia is an abstract concept. It simply gives a measure of rotational inertia of a rigid body and it is proportional to the product of the square of radius, r of the body and its mass, m . Students by referring to websites, can construct and perform simple experiments to verify that $MI \propto mr^2$. Reference : www.khanacademy.org , www.pinterest.com , www.serc.cerleton.edu	
Activity No. 4	Activity: Prepare suitable charts and give seminar talks in the class.	

Unit - 3		
Chapter No. 7	Elasticity: Hooke's law - Stress-strain diagram, elastic moduli- relation between elastic constants, Poisson's Ratio-expression for Poisson's ratio in terms of elastic constants. Work done in stretching and work done in twisting a wire- Twisting couple on a cylinder. Torsional pendulum-Determination of rigidity modulus and moment of inertia - q , η and σ by Searle's method	(13)
	Suggested Activities	

Activity No. 5	Activity: Arrange a steel spring with its top fixed with a rigid support on a wall and a meter scale along side. Add 100 g load at a time on the bottom of the hanger in steps. This means that while putting each 100g load, we are increasing the stretching force by 1N. Measure the extension for loads up to 500g. Plot a graph of extension versus load. Shape of the graph should be a straight line indicating that the ratio of load to extension is constant. Go for higher loads and find out elastic limit of the material.	
Activity No.6	Activity: Repeat the above experiment with rubber and other materials and find out what happens after exceeding elastic limit. Plot and interpret.	

Unit - 4		
Chapter No. 8	Surface tension: Definition of surface tension. Surface energy, relation between surface tension and surface energy, pressure difference across curved surface example, excess pressure inside spherical liquid drop, angle of contact.	(13)
Chapter No.9	Viscosity: Streamline flow, turbulent flow, equation of continuity, determination of coefficient of viscosity by Poissulle's method, Stoke's method. Problems.	
Topics for self study(If any)	Capillarity determination of surface tension by drop weight method. Ref: 6,7,9,10	
	Suggested Activities	
Activity No.7	1. Measure surface tension of water and other common liquids and compare and learn i) Why water has high ST? think of reasons. ii) Check whether ST is a function of temperature? You can do it by heating the water to different temperatures and measure ST. iii) Plot ST versus T and learn how it behaves. Mix some quantity of kerosene or any oil to water and measure ST. Check whether ST for the mixture is more or less than pure water. List the reasons.	

Activity No. 8	Activity: 2) Collect a set of different liquids and measure their viscosity. 3) Find out whether sticky or non-sticky liquids are most viscous. List the reasons. ii) Mix non sticky liquid to the sticky liquid in defined quantities and measure viscosity. Find out viscosity is increasing or decreasing with increase of non-sticky liquid concentration. iii) Do the above experiment by mixing sticky liquid to the non sticky liquid. Find out change in viscosity with increase of concentration of sticky liquid. List the applications where concept of Viscosity plays a dominant role	
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Text Books:

Sl No	Title of the Book	Authors Name	Publisher	Year of Publication
1	Mechanics by, New Edition	D. S. Mathur	S.Chand & Co.	2000
2	Mechanics and Relativity by 3 rd Edition,	Vidwan Singh Soni,	PHI Learning Pvt. Ltd.	
3	Mechanics Berkeley Physics Course, Vol.1:	Charles Kittel, <i>et.al.</i>	Tata McGraw-Hill	2007
4	Properties of Matter	Brijlal & Subramanyam.		

References/Books

Sl No	Title of the Book	Authors Name	Publisher	Year of Publication
1	Physics. 9 th Edn,	Resnick, Halliday & Walter,	Wiley	2010
2	Physics Vol-I	Halliday and Resnick,		

List of Experiments to be performed in the Laboratory:

(Minimum EIGHT experiments have to be carried out)

1.	Determination of g using bar pendulum (L versus T and L versus LT^2 graphs).
2.	Determination of moment of inertia of a Fly Wheel.
3.	Determination of rigidity modulus using torsional pendulum.
4.	Modulus of rigidity of a rod – Static torsion method.
5.	Determination of elastic constants of a wire by Searle's method.
6.	Young's modulus by Koenig's method.
7.	Viscosity by Stoke's method.
8.	Verification of Hook's law.
9.	Determination of surface tension of a liquid and the interfacial tension between two liquids using drop weight method.
10.	Study of motion of a spring and to calculate Spring constant, g and unknown mass.
11.	Determination of Young's modulus of a bar by the single cantilever method.
12.	Determination of Young's modulus of a bar by uniform bending method.
13.	Radius of capillary tube by mercury pellet method.
14.	Verification of parallel and perpendicular axis theorems.
15.	M.I. of an irregular body
16.	Determination of Young's modulus, rigidity modulus and ρ - Searl's double bar

Reference Book for Laboratory Experiments

Sl No	Title of the Book	Authors Name	Publisher	Year of Publication
1	Physics through experiments	B.Saraf	Vikas Publications	2013
2	A lab manual of Physics for undergraduate classes, 1 st Edition,		Vikas Publications.	
3	BSc Practical Physics Revised Ed	CL Arora	S.Chand & Co.	2007
4	An advanced course in practical physics.	D. Chatopadhyay, PC Rakshit, B.Saha	New Central Book Agency Pvt Ltd.	2002

Formative Assessment	
Assessment	Marks
Internal Assessment	10
Activity	10
REU based Group Activity (Conduction, Report, Presentation)	10
Science Communication (Seminar/Poster/ Mini Project etc)	10
Total	40

II Semester (Core)

Electricity & Magnetism

Course Title: Electricity and Magnetism	Course Credits: 4
Total Contact Hours: 52	Duration of ESA: 2 hours
Formative Assessment Marks: 40	Summative Assessment Marks: 60
Model Syllabus Authors:	Physics Expert Committee

Electricity & Magnetism		Hrs
Unit – 1		
Chapter No. 1	Topics to be covered/taught/learned: Electric charge and field, Coulomb's law, electric field strength, electric field lines, point charge in an electric field and electric dipole, work done by a charge (derivation of the expression for potential energy)	3
Chapter No. 2	Topics to be Covered Gauss's law and its applications (electric fields of a (i) spherical charge distribution, (ii) line charge and (iii) an infinite flat sheet of charge).	3
Chapter No. 3	Topics to be Covered Electric potential, line integral, gradient of a scalar function, relation between field and potential. Potential due to point charge and distribution of charges (Examples: potential associated with a spherical charge distribution, infinite line charge distribution, infinite plane sheet of charges). Constant potential surfaces, Potential due to a dipole and electric quadrupole.	7
Topics for self study(If any)	<i>Constant potential surfaces - for self learning</i> <i>Work out problems listed in the reference</i>	
Suggested Activities		
Activity No. 1	1. Learn the difference between DC and AC electricity and their characteristics. Voltage and line frequency standards in different countries. 2. A small project report on production of electricity as a source of energy: Different methods	
Activity No. 2	1. Learn to use a multimeter (analog and digital) to measure voltage, current and resistance. Continuity testing of a wire. 2. Learn about household electrical connection terminals: Live, neutral and ground and voltage between the terminals. Role of earthing and safety measures	

Unit – 2		
Chapter No. 4.	Topics to be covered Conductors in electrostatic field Conductors and insulators, conductors in electric field. Capacitance and capacitors, calculating capacitance in a parallel plate capacitor, parallel plate capacitor with dielectric, dielectrics: an atomic view. Energy stored in a capacitor, Dielectric and Gauss's law.	6
Chapter No. 5.	Topics to be covered Electric currents and current density. Electrical conductivity and Ohm's law. Physics of electrical conduction, conduction in metals and semiconductors, circuits and circuit elements: Variable currents in capacitor circuits, Resistor, inductor and capacitor and their combination. force on a moving charge.	7
Topics for self study(If any)	<i>Currents and voltage in combination of R, L and C circuits</i>	
	Suggested Activities	
Activity No. 3	1. Learn about electrical appliances which work with AC and DC electricity 2. Learn about types of resistors and their colour codes and types of capacitors (electrolytic and non-electrolytic)	
Activity No. 4	1. Learn about power transmission: 3-phase electricity, voltage and phase 2. Visit a nearby electrical power station. Interact with line men, Electrical engineers and managers. Discuss about power loss in transmission. How to reduce it? 3. Prepare a small project report on street lighting and types of electrical bulbs.	
Unit – 3		
Chapter No.6	Topics to be covered Magnetism: Definition of magnetic field, Ampere's law and Biot-Savart law (magnetic force and magnetic flux), Magnetic force on a current carrying conductor, Hall effect. Electromagnetic induction, conducting rod moving in a magnetic field, law of induction and mutual inductance, self inductance and energy stored in a magnetic field.	7
Chapter No. 7	Topics to be covered Alternating current circuits: Resonant circuit, alternating current, quality factor, RL, RC, LC, LCR circuits, admittance and impedance, power and energy in AC circuits.	6
Topics for self study(If any)	Hall Effect	

	Suggested Activities	
Activity No. 5	Activity: 1. Prepare a small project report on street lighting and types of electrical bulbs. 2. Learn the measurement of electric current using tangent galvanometer.	
Activity No.6	Activity: Build a small coil with insulated copper wire. Connect an ammeter micro/milli ammeter. Verify magnetic induction using a powerful bar magnet.	
Unit - 4		
Chapter No. 8	Electromagnetic waves: Equation of continuity, Maxwell's equations, displacement current, electromagnetic wave, energy transported by electromagnetic waves. Electromagnetic waves in different frames of reference, Field of a current loop, magnetic moment, Electric current in atoms, electron spin and magnetic moment, magnetization and magnetic susceptibility.	8
Chapter No. 9	Topics to be covered: Types of magnetic materials: diamagnetic, paramagnetic and ferromagnetic materials. B-H hysteresis curves.	5
Topics for self study(If any)	<i>B-H curves and its characteristics</i> <i>Ferrites</i>	
	Suggested Activities	
Activity No.7	Activity: 1. Prepare a small project report on production of magnetic field: Permanent magnets, electromagnets and superconducting magnets. 2. Learn the principle of working of a Gauss meter to measure magnetic field	
Activity No. 8	Activity: 1. Model the earth's magnetic field with a diagram. Explain the effect of tilt of the earth's axis and reasons for the change in the tilt of the earth's axis over thousands of years.	

References Books:

SINo	Title of the Book	Authors Name	Publisher	Year of Publication
1	Physics-Part-II,	David Halliday and Robert Resnick	Wiley Eastern Limited	2001
2	Berkeley Physics Course, Vol-2, Electricity and Magnetism, Special Edition	Edward M Purcell	Tata Mc Graw-Hill Publishing Company, Ltd, New Delhi	2008

List of Experiments to be performed in the Laboratory:
(Minimum EIGHT Experiments to be carried out)

1.	Experiments on tracing of electric and magnetic flux lines for standard configuration.
2.	Determination of components of earth's magnetic field using a Ballistic galvanometer.
3.	Determination of capacitance of a condenser using B.G.
4.	Determination of high resistance by leakage using B.G.
5.	Determination of mutual inductance using BG.
6.	Charging and discharging of a capacitor (energy dissipated during charging and time constant measurements).
7.	Series and parallel resonance circuits (LCR circuits).
8.	Impedance of series RC circuits- determination of frequency of AC.
9.	Study the characteristics of a series RC and RL Circuit.
10.	Determination of self inductance of a coil-Anderson's Bridge
11.	Verification of laws of combination of capacitances and determination of unknown capacitance using de - Sauty bridge.
12.	Determination of B_H using Helmholtz double coil galvanometer and potentiometer.
13.	Variation of X_c with frequency and determination of Capacitance.
14.	Measurement of low resistance of a coil using potentiometer

Formative Assessment	
Assessment	Marks
Internal Assessment	10
Activity	10
REU based Group Activity (Conduction, Report, Presentation)	10
Science Communication (Seminar/Poster/ Mini Project etc)	10
Total	40

III Semester (Core)

Wave Motion and Optics

Program Outcomes:	
1.	Disciplinary knowledge
2.	Communication Skills
3.	Critical thinking, Reflective thinking, Analytical reasoning, Scientific reasoning
4.	Problem-solving
5.	Research-related skills
6.	Cooperation/ Teamwork/ Leadership readiness/Qualities
7.	Information/ Digital literacy/Modern Tool Usage
8.	Environment and Sustainability
9.	Multicultural competence
10.	Multi-Disciplinary
11.	Moral and ethical awareness/Reasoning
12.	Lifelong learning / Self Directed Learning

Course Content Semester – III	
Course Title: Wave Motion and Optics	Course Credits: 4
Total Contact Hours: 52	Duration of ESA: 2 hours
Formative Assessment Marks: 40	Summative Assessment Marks: 60
Model Syllabus Authors: Physics Expert Committee	

Prerequisites	
i.	Fundamentals of waves

Course Learning Outcomes	
At the end of the course students will be able to:	
i.	Identify different types of waves by looking into their characteristics.
ii.	Formulate a wave equation and obtain the expression for different parameters associated with waves.
iii.	Explain and give a mathematical treatment of the superposition of waves under different conditions, such as, when they overlap linearly and perpendicularly with equal or different frequencies and equal or different phases.
iv.	Describe the formation of standing waves and how the energy is transferred along the standing wave in different applications, and mathematically model in the case of stretched string and vibration of a rod.
v.	Give an analytical treatment of resonance in the case of open and closed pipes in general and Helmholtz resonators in particular.
vi.	Describe the different parameters that affect the acoustics in a building, measure it and control it.
vii.	Give the different models of light propagation and phenomenon associated and measure the parameters like the wavelength of light using experiments like Michelson interferometer, interference and thin films.
viii.	Explain diffraction due to different objects like singles slit, two slits, diffraction of grating, oblique incidence, circular aperture and give the theory and experimental setup for the same.
ix.	Explain the polarization of light and obtain how the polarization occurs due to quarter wave plates, half wave plates, and through the optical activity of a medium.

Course Articulation Matrix													
Mapping of Course Outcomes (CO) Program Outcomes													
Course Outcomes / Program Outcomes		1	2	3	4	5	6	7	8	9	10	11	12
i.	Identify different types of waves by looking into their characteristics.	X	X	X	X	X	X					X	X
ii.	Formulate a wave equation and obtain the expression for different parameters associated with waves.	X	X	X	X	X	X					X	X
iii.	Explain and give a mathematical treatment of the superposition of waves under different conditions such as when they overlap linearly and perpendicularly	X	X	X	X	X	X					X	X

	with equal or different frequencies and equal or different phases.												
iv.	Describe the formation of standing waves and how the energy is transferred along the standing wave in different applications, and mathematically model in the case of stretched string and vibration of a rod.	X	X	X	X	X	X					X	X
v.	Give an analytical treatment of resonance in the case of open and closed pipes in general and Helmholtz resonators in particular.	X	X	X	X	X	X					X	X
vi.	Describe the different parameters that affect the acoustics in a building, measure it and control it.	X	X	X	X	X	X					X	X
vii.	Give the different models of light propagation and phenomenon associated and measure the parameters like the wavelength of light using experiments like Michelson interferometer, interference and thin films.	X	X	X	X	X	X					X	X
viii.	Explain diffraction due to different objects like single slit, two slits, diffraction grating, oblique incidence, circular aperture and give the theory and experimental setup for the same.	X	X	X	X	X	X					X	X
ix.	Explain the polarization of light and obtain how the polarization occurs due to quarter wave plates, half wave plates, and through the optical activity of a medium.	X	X	X	X	X	X					X	X

Wave Motion and Optics				
Unit – 1 – Waves and Superposition of Harmonic Waves				
The Portion to be Covered				
Waves: Plane and Spherical Waves. Longitudinal and Transverse Waves. Characteristics of wave motion, Plane Progressive (Travelling) Wave and its equation, Wave Equation – Differential form (derivation). Particle and Wave Velocities: Relation between them, Energy Transport – Expression for intensity of progressive wave, Newton's Formula for Velocity of Sound. Laplace's Correction (Derivation). Brief account of Ripple and Gravity Waves. (Text Book : 1-4) (5 Hours) Superposition of Harmonic Waves : Linearity and Superposition Principle. Superposition of two collinear oscillations having (1) equal frequencies and (2) different frequencies (Beats) – Analytical treatment. Superposition of two perpendicular Harmonic Oscillations: Lissajous Figures with equal and unequal frequency- Analytical treatment. Uses of Lissajous' figures. (Text Book : 1-4) (6 Hours)				
Topic Learning Outcomes At the end of the topic, students should be able to:				
SL No	TLO's	BL	CO	PO
i.	Explain the difference between plane and spherical waves, longitudinal and transverse waves and give their characteristics.	L2	1	1-6, 11-12
ii.	Write down an equation for the progressive wave in its differential form.	L2	1	1-6, 11-12
iii.	Obtain the relation between particle and wave velocity.	L2	1	1-6, 11-12
iv.	Obtain an expression for intensity of progressive waves.	L2	1	1-6, 11-12
v.	Obtain Newton's formula for the velocity of sound and discuss the factors for which sound velocity is dependent.	L2	2	1-6, 11-12
vi.	Apply the Laplace's correction to the equation of motion of a progressive wave.	L2	2	1-6, 11-12
vii.	With examples explain ripple and gravity waves.	L1	2	1-6, 11-12
viii.	Give the theory of superposition of two linear waves having equal frequencies and different frequencies.	L2	3	1-6, 11-12
ix.	Discuss the formation of different Lissajous figures under different conditions of amplitude and frequency when they superimpose perpendicularly.	L2	3	1-6, 11-12
x.	Give some applications of an Lissajous figures.	L1	3	1-6, 11-12
xi.	Higher order problems.	L3	1,2,3	1-6, 11-12

Teaching and Learning Methodology	
Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.	
Assessment Techniques	
One minute paper/ Predict-Observe-Explain/ Think-Pair-Share/ Class Test/ Quiz/ Crosswords/ Group Assessment/ Assignment/ Peer-to-Peer Evaluation/Seminar etc	
Suggested Activities (2 Hours)	
Activity No. 1	We know that sound is produced because of vibration. Look into at least 10 musical instruments and identify the regions of vibrations that produces the sound and those parts which enhances the sound because of reverberation. 1. Identify one common element in all of these. 2. Identify equipment which creates beats and try to explain the underlying basic principles. Demonstrate the examples of beats using two tuning forks. 3. Identify what will happen when you drop a stone in a standing water, and when your drop two stones side by side. 4. Make your observations sketch them and comment on it in a report.
Activity No. 2	Draw two sine waves (Amplitude vs time) one shifted with other in phase. Identity where the resonance occurs for each phase shift. Plot phase vs time taken for resonance.
Activity No. 3	Take smooth sand, place a pointed edged pen vertically on the sand. To the mid of the pen, connect two perpendicular threads. Pull these perpendicular threads by varying the forces and timings. Note down the different shapes produced on the sand. Try to interpret the shapes. Make a report of it
Activity No. 4	Hang a pot with sand, which has a hole in the bottom. Gently pull the pot on one side and observe the pattern formed by the sand on the floor. Report the observations.
Activity No. 5	Design a coupled pendulum. Study the impact of the motion of one pendulum over the other pendulum by varying the length, direction of the motion of one pendulum and mass of pendulum and observe the resultant changes. Trace the path of the bobs and make a report.
Activity No. 6	Note for the teachers for the activity: Make 3 groups among students and assign each group the activity of drawing one of the 3 graphs given below. Provide a few days to complete the activity. One the specific day, each group has to make a ppt presentation of the following three slides. One the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation,

	teacher shall assign marks to each group, wherein all members of the group will get equal marks. 1. The first slide will explain the process of doing the experiment. 2. In the second slide. Students will show the graph of measurement. 3. In the third slide, they will list three observations from that study. Activity: Take a stretched spring. Stretch it across two edges. Put a weight on the string, pluck it and measure the amplitude of the vibration. All group will measure the total damping time of oscillating spring. (Using mobile or scale) And plot a graph of the- 1. Varying load on the spring and amplitude at the centre. 2. Take another weight and put that in another place and measure the amplitude of vibration at the centre. 3. Vary the load in the centre of the spring and measure the amplitude at the centre.
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Wave Motion and Optics				
Unit – 2 – Standing Waves and Acoustics				
The Portion to be Covered				
Standing Waves : Velocity of transverse waves along a stretched string (derivation), Standing (Stationary) Waves in a String – Fixed and Free Ends (qualitative). Theory of Normal modes of vibration in a stretched string, Energy density and energy transport of a transverse wave along a stretched string. Vibrations in rods – longitudinal and transverse modes (qualitative). Velocity of Longitudinal Waves in gases (derivation). Normal Modes of vibrations in Open and Closed Pipes – Analytical treatment. Concept of Resonance, Theory of Helmholtz resonator. (Text Book : 1-4) (8 Hours) Acoustics: Absorption coefficient, Reverberation and Reverberation time, Sabine's Reverberation formula (derivation), Factors affecting acoustics in buildings, Requisites for good acoustics. Acoustic measurements – intensity and pressure levels. (Text Book : 1-4) (3 Hours)				
Topic Learning Outcomes At the end of the topic, students should be able to:				
SL No	TLO's	BL	CO	PO
i.	Discuss the Transverse waves produced in stretched string and obtain the expression for the same.	L2	3	1-6, 11-12
ii.	Give a qualitative treatment of vibration of a string when it's both ends are fixed and free.	L2	3	1-6, 11-12

iii.	Explain normal modes of a stretched string. Obtain an expression for the energy density and discuss how this energy is transported along a stretched string.	L2	3	1-6, 11-12
iv.	Quantitatively bring about the mode of vibrations created in a rod.	L2	4	1-6, 11-12
v.	Explain types of waves that are produced in gas. Obtain an expression for the same.	L2	4	1-6, 11-12
vi.	With an analytical treatment explain the concept of resonance using the normal modes of vibrations of open and closed pipes.	L2	5	1-6, 11-12
vii.	Give the theory of Helmholtz resonator and explain how it is used to calculate some parameters of the way the standing waves are set in there.	L2	5	1-6, 11-12
viii.	Define Reverberation, Reverberation time and absorption coefficient of a material.	L1	5	1-6, 11-12
ix.	Obtain Sabine's Reverberation formula and discuss what are the factors on which the Reverberation time depends on?	L2	5	1-6, 11-12
x.	List out which are different parameters within a building which effects the acoustics.	L1	6	1-6, 11-12
xi.	Explain what good acoustics of a building are and how acoustics is measured in terms of intensity and pressure inside a building.	L2	6	1-6, 11-12
xii.	Higher order problems.	L3	4,5,6	1-6, 11-12
Teaching and Learning Methodology				
Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.				
Formative Assessment Techniques				
One minute paper/ Predict-Observe-Explain/ Think-Pair-Share/ Class Test/ Quiz/ Crosswords/ Group Assessment/ Assignment/ Peer-to-Peer Evaluation/Seminar etc.				

Suggested Activities (2 Hours)	
Activity No. 7	List different phenomenon where standing waves are found in nature. Identify the phenomena and reason for standing waves. Also identify the standing waves in musical instruments. Make a report.
Activity No. 8	1. Go to 5 different newly constructed houses when they are not occupied and when they are occupied. Make your observations on sound profile on each room. Give the reasons. Make a report. 2. Visit three very good auditoriums, list out different ways in which the acoustic arrangements have been done (as decoration and Civil works). Look for the reasons in Google and identify which is acoustically the best auditorium among the three you visited. Make a report.
Activity No. 9	Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. One the specific day, each group has to make a ppt presentation of the following three slides. One the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. 1. The first slide will explain the process of doing the experiment. 2. In the second slide. Students will show the graph of measurement. 3. In the third slide, they will list three observations from that study. Activity: Take a bowl of different liquids (water, milk, kerosene, salt water, Potassium Permanganate (KMNO₄) solution. Place a small non oily floating material (ex: thin plastic) on the surface of the liquid. Drop a marble on the liquid at the centre of the bowl. Repeat the experiment by dropping the marble from the different heights. Plot a graph of- 1. Height v/s time of oscillation 2. Weight of the marble v/s time of oscillation
Activity No. 10	Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. One the specific day, each group has to make a ppt presentation of the following three slides. One the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. 1. The first slide will explain the process of doing the experiment. 2. In the second slide. Students will show the graph of measurement. 3. In the third slide, they will list three observations from that study.

	Activity: Take two marble of same weight. Drop both the marbles on the surface of the liquid from some height. With the help of the mobile take the picture and measure the position of interface of two wave fronts formed in the liquid. Plot graphs for different activities by doing the following activities. 1. By dropping two marbles of same weight from different heights. 2. By dropping two marbles of different weight from the same height
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Wave Motion and Optics				
Unit – 3 – Nature of light and Interference				
The Portion to be Covered				
Nature of light : The corpuscular model of light-The wave model – Maxwells electromagnetic waves- Wave Particle Duality (Text Book No 5; Sections 2.1 to 2.4 and 2.8) (2 Hours) Interference of light by division of wave front: Huygen’s theory-Concept of wave-front-Interference pattern produced on the surface of water-Coherence-Interference of light waves by division of wave-front- Young’s double slit experiment- derivation of expression for fringe width-Fresnel Biprism-Interference with white light (Text Book No 5; Sections 12.1 to 12.2, 14.1 to 14.5, 14.7 to 14.9) (4 Hours) Interference of light by division of amplitude: Interference by division of amplitude-Interference by a plane parallel film illuminated by a plane wave-Interference by a film with two non-parallel reflecting surfaces- color of thin films—Newton’s rings-(Reflected light)-Michelson Interferometer-Determination of wavelength of light* (Text Book No 5; Sections 15.1 to 15.2, 15.8 to 15.11) (5 Hours)				
Topic Learning Outcomes At the end of the topic, students should be able to:				
SL No	TLO’s	BL	CO	PO
i.	Explain using Michelson interferometer how to determine the wavelength of light.	L2	7	1-6, 11-12
ii.	Give an account of the different possible shapes that are obtained in Michelson interferometer experiment and their relevance.	L2	7	1-6, 11-12
iii.	Discuss the wave model and the Corpuscular model of light.	L2	7	1-6, 11-12
iv.	Explain Maxwells electromagnetic waves.	L2	7	1-6, 11-12
v.	Give an account of the phenomenon of wave-particle duality.	L1	7	1-6, 11-12
vi.	Give the Huygen theory of wave-front.	L1	7	1-6, 11-12

vii.	Define Interference. Give some examples of Interference.	L1	7	1-6, 11-12
viii.	Give the theory of interference due to two coherent sources of light and obtain an expression for the wavelength of monochromatic source of light (Young's double slit experiment)	L2	7	1-6, 11-12
ix.	Explain how using personal biprism, a monochromatic coherent source of light are obtained. Using this experimental setup explain how the wavelength of monochromatic sources of light is determined.	L2	7	1-6, 11-12
x.	Give the theory of interference due to division of amplitude by parallel and non-parallel plates.	L1	7	1-6, 11-12
xi.	Explain how Newton's rings are obtained and discuss how the wavelength of light is determined using this experiment.	L2	7	1-6, 11-12
xii.	Higher order problems.	L3	7	1-6, 11-12
Teaching and Learning Methodology				
Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.				
Formative Assessment Techniques				
One minute paper/ Predict-Observe-Explain/ Think-Pair-Share/ Class Test/ Quiz/ Crosswords/ Group Assessment/ Assignment/ Peer-to-Peer Evaluation/Seminar etc				

Suggested Activities (2 Hours)

Activity No. 11	In the table given below explore which phenomenon can be explained by what and Make a report. <table><tr><th>Sl No</th><th>Phenomenon</th><th>Particle of Light</th><th>Wave Nature</th><th>Dual Nature</th></tr><tr><td></td><td>Pinhole camera</td><td></td><td></td><td></td></tr><tr><td>1</td><td>Formation of images on lenses</td><td></td><td></td><td></td></tr><tr><td>2</td><td>Formation of images on mirror</td><td></td><td></td><td></td></tr><tr><td>3</td><td>Interference</td><td></td><td></td><td></td></tr><tr><td>4</td><td>Polarization</td><td></td><td></td><td></td></tr><tr><td>5</td><td>Diffraction due to single slit</td><td></td><td></td><td></td></tr><tr><td>6</td><td>Black body radiation</td><td></td><td></td><td></td></tr><tr><td>7</td><td>Photoelectric effect</td><td></td><td></td><td></td></tr><tr><td>8</td><td>De-Broglie hypothesis</td><td></td><td></td><td></td></tr><tr><td>9</td><td>Devison & Germer</td><td></td><td></td><td></td></tr></table>	Sl No	Phenomenon	Particle of Light	Wave Nature	Dual Nature		Pinhole camera				1	Formation of images on lenses				2	Formation of images on mirror				3	Interference				4	Polarization				5	Diffraction due to single slit				6	Black body radiation				7	Photoelectric effect				8	De-Broglie hypothesis				9	Devison & Germer			
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Activity No. 12	Why colour strips are seen in paddles on roads in rainy seasons try to simulate the same. Give the reasons. Make a report.																																																							
Activity No. 13	Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. One the specific day, each group has to make a ppt presentation of the following three slides. One the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. 1. The first slide will explain the process of doing the experiment. 2. In the second slide. Students will show the graph of measurement. 3. In the third slide, they will list three observations from that study. Activity: Take a bowl of different liquids (water, milk, kerosene, salt water, Potassium Permanganate (KMNO4) solution. Place a small non oily floating material (ex: thin plastic) on the surface of the liquid. Drop two marbles of same weight (mass) from the same height on to the surface of the water but at the different time intervals. Plot graph for the different observations. For teachers: Demonstrate the formation of Lissajous Figure using a CRO. Give different shapes of Lissajous Figure with varying frequency and amplitude. Ask the students to comment on the observations.																																																							

Wave Motion and Optics				
Unit – 4 – Diffraction and Polarisation				
The Portion to be Covered				
Fraunhofer diffraction : Introduction- Fraunhofer diffraction- Single slit diffraction pattern-position of Maxima and Minima (Qualitative arguments)- Two slit diffraction pattern-position of Maxima and minima- Theory of plane diffraction Grating-Grating spectrum- normal and oblique incidence- Resolving power and dispersive power of a grating Single slit; Double Slit. Multiple slits & Diffraction grating. (Text Book No 5; Sections 18.1 to 18.2, 18.6,18.8 to 18.9) (4 Hours) Fresnel Diffraction- Fresnel half period zones-Diffraction by a circular aperture-diffraction by an opaque disc-The zone plate –comparison between zone plate and convex lens. (Text Book No 5; Sections 20.1 to 20.3) (3 Hours) Polarisation: Introduction-Production of polarized light- The wire Grid polarizer and Polaroid-Superposition of two disturbances-Phenomenon of double refraction-Quarter wave plates and half wave plates- Analysis of polarized light-optical activity. (Text Book No 5; Sections 22.1, 22.3,22.4,22.6 to 22.8) (4 Hours)				
Topic Learning Outcomes At the end of the topic, students should be able to:				
SL No	TLO's	BL	CO	PO
i.	Define Fraunhofer diffraction.	L2	8	1-6, 11-12
ii.	Give a qualitative treatment of single slit/diffraction double slit diffraction.	L2	8	1-6, 11-12
iii.	Explain the theory of diffraction due to grating and the normal and oblique incidence.	L2	8	1-6, 11-12
iv.	Explain how the resolving power of a grating depends of the number of slits used.	L2	8	1-6, 11-12
v.	Give the theory of Fresnel half period zones.	L2	8	1-6, 11-12
vi.	Discuss zone plates with respect to convex lenses.	L2	8	1-6, 11-12
vii.	Explain optical polarization and polaroids.	L2	9	1-6, 11-12
viii.	Give different types of polaroids.	L2	9	1-6, 11-12
ix.	Give the theory of phenomenon of double refraction and explain what are ordinary and extraordinary rays.	L2	9	1-6, 11-12
x.	Give the theory of quarter wave plates and half wave plates.	L2	9	1-6, 11-12
xi.	Explain optical activity with theory. Give an experimental method to measure the optical activity of a material.	L2	9	1-6, 11-12
xii.	Higher order problems.	L3	8,9	1-6, 11-12

Teaching and Learning Methodology	
Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.	
Assessment Techniques	
One minute paper/ Predict-Observe-Explain/ Think-Pair-Share/ Class Test/ Quiz/ Crosswords/ Group Assessment/ Assignment/ Peer-to-Peer Evaluation/Seminar etc	
Suggested Activities (2 Hours)	
Activity No. 14	Explain polarization of light through a chart. List out the surfaces that reflect polarized light. Learn how polarization of light can be done by both transmission and reflection. Perform an experiment and make a report. USING CDs AND DVDs AS DIFFRACTION Gratings Ref:https://www.nnin.org/sites/default/files/files/Karen_Rama_USING_CDs_AND_DVDs_AS_DIFFRACTION_GRATINGS_0.pdf Obtain the diffraction spectra using a CD and design an experiment to find the distance between the tracks on it) (Ref: https://www.brighthubeducation.com/science-lessons-grades-9-12/39347-diffraction-experiment-measuring-groove-spacing-on-cds/, https://silo.tips/download/diffraction-from-a-compact-disk)
Activity No. 15	What is the physics behind making 3D movies? Group Discussion (https://www.slideserve.com/rae/physics-behind-3d-movies-powerpoint-ppt-presentation) Make a report.
Activity No. 16	List out different types of zone plates and look for their applications in day to day life. Make a report.
Activity No. 17	Collect information and study how optically polarizing lenses are made. Visit a nearby lens making facility. Learn the principle behind sunglasses. Make a report.
Activity No. 18	Note for the teachers for the activity: Make 3 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. One the specific day, each group has to make a ppt presentation of the following three slides. One the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. 1. The first slide will explain the process of doing the experiment.

	2. In the second slide. Students will show the graph of measurement. 3. In the third slide, they will list three observations from that study. Activity: Identify any 3 sharp edges of varying thickness and assign them to 3 groups. Shine a laser light pointing towards the edge of the needle. Observe the patterns formed on the wall or screen and measure the distance between the bands. Correlate the distance between the bands formed with the thickness of the edge and the distance from the edge to the screen. By this, calculate the wavelength of the laser light used.
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Textbooks				
SI No	Title of the Book	Authors Name	Publisher	Year of Publication
1.	The Physics of Waves and Oscillations,	N K Bajaj	Tata McGraw-Hill Publishing Company Ltd., Second Edition,	1984
2.	Waves and Oscillations	N Subramanyam and Brij Lal	Vikas Publishing House Pvt. Ltd., Second Revised Edition	2010
3.	A Text Book of Sound	D R Khanna and R S Bedi	Atma Ram & Sons, Third Edition	1952
4.	Oscillations and Waves	Satya Prakash	Pragathi Prakashan, Meerut, Second Edition	2003
5.	Optics	Ajoy Ghatak	McGraw Hill Education (India) Pvt Ltd	2017
6.	A text Book of Optics	Brij Lal, M N Avadhanulu & N Subrahmanyam	S. Chand Publishing	2012

References Books				
SI No	Title of the Book	Authors Name	Publisher	Year of Publication
1.	Berkeley Physics Course – Waves,	Frank S Crawford Jr.	Tata Mc Graw-Hill Publishing Company Ltd., Special Indian Edition,.	2011
2.	Optics	Eugene Hecht	Pearson Paperback	2019

3.	Introduction To Optics	Pedrotti and Frank L ,	Pearson India	3 rd Edition
4.	Fundamentals of Optics	Francis Jenkins Harvey White	McGraw Hill Education	2017

Formative Assessment	
Assessment	Marks
Internal Assessment	10
Activity	10
REU based Group Activity (Conduct, Report, Presentation)	10
Science Communication Seminar/Poster etc.)	10
Total	40

List of Experiments to be performed in the Laboratory (Minimum EIGHT Experiments have to be carried out)	
1.	Velocity of sound through a wire using Sonometer.
2.	Frequency of AC using Sonometer.
3.	Study of Lissajous' Figures
4.	To verify the laws of transverse vibration using Melde's apparatus.
5.	Helmholtz resonator using tuning fork.
6.	Helmholtz resonator using electrical signal generator.
7.	To determine refractive index of the Material of a prism (Calcite) using sodium source.
8.	To determine the dispersive power and Cauchy constants of the material of a prism using mercury source.
9.	To determine the wavelength of sodium source using Michelson's interferometer.
10.	To determine wavelength of sodium light using Fresnel Biprism.
11.	To determine wavelength of sodium light using Newton's Rings
12.	To determine the thickness of a thin paper by measuring the width of the interference fringes produced by a wedge-shaped Film.
13.	To determine wavelength of (1) Na source and (2) spectral lines of Hg source using plane diffraction grating.
14.	To determine dispersive power and resolving power of a plane diffraction grating.
15.	To determine Specific rotation of a given solution – Using Polarimeter

Reference Book for Laboratory Experiments				
SI No	Title of the Book	Authors Name	Publisher	Year of Publication
1.	Advanced Practical Physics for students	B.L. Flint and H.T. Worsnop	Asia Publishing House.	1971
2.	A Text Book of Practical Physics	1. Prakash & Ramakrishna	Kitab Mahal, 11 th Edition	2011
3.	Advanced level Physics Practicals	Michael Nelson and Jon M. Ogborn	Heinemann Educational Publishers, 4 th Edition	1985
4.	A Laboratory Manual of Physics for undergraduate classes	D.P.Khandelwal	Vani Publications.	1985

IV Semester (Core)

Thermal Physics and Electronics

Program Outcomes:	
1.	Disciplinary knowledge
2.	Communication Skills
3.	Critical thinking, Reflective thinking, Analytical reasoning, Scientific reasoning
4.	Problem-solving
5.	Research-related skills
6.	Cooperation/ Teamwork/ Leadership readiness/Qualities
7.	Information/ Digital literacy/Modern Tool Usage
8.	Environment and Sustainability
9.	Multicultural competence
10.	Multi-Disciplinary
11.	Moral and ethical awareness/Reasoning
12.	Lifelong learning / Self Directed Learning

Course Content Semester – IV	
Course Title: Thermal Physics and Electronics	Course Credits: 4
Total Contact Hours: 52	Duration of ESA: 2 hours
Formative Assessment Marks: 40	Summative Assessment Marks: 60
Model Syllabus Authors: Physics Expert Committee	

Prerequisites	
i.	Study of Pre-University

Course Learning Outcomes	
At the end of the course students will be able to:	
i.	Apply the laws of thermodynamics and analyze the thermal system.
ii.	Apply the laws of kinetic theory and radiation laws to the ideal and practical thermodynamics systems through derived thermodynamic relations.
iii.	Use the concepts of semiconductors to describe different Semiconductor devices such as diode transistors, BJT, FET etc and explain their functioning.
iv.	Explain the functioning of OP-AMPS and use them as the building blocks of logic gates.
v.	Give the use of logic gates using different theorems of Boolean Algebra followed by logic circuits.

Course Articulation Matrix													
Mapping of Course Outcomes (CO) Program Outcomes													
Course Outcomes / Program Outcomes		1	2	3	4	5	6	7	8	9	10	11	12
i.	Apply the laws of thermodynamics and analyze the thermal system.	X	X	X	X	X	X					X	X
ii.	Apply the laws of kinetic theory and radiation laws to the ideal and practical thermodynamics systems through derived thermodynamic relations.	X	X	X	X	X	X					X	X
iii.	Use the concepts of semiconductors to describe different Semiconductor devices like diode transistors, BJT, FET etc and explain their functioning.	X	X	X	X	X	X					X	X
iv.	Explain the functioning of OP-AMPS and them as the building blocks of logic gates.	X	X	X	X	X	X					X	X
v.	Give the use of logic gates using different theorems of Boolean Algebra followed by logic circuits.	X	X	X	X	X	X					X	X

Thermal Physics and Electronics				
Unit – 1				
The Portion to be Covered				
Laws of Thermodynamics: Review of the concepts of Heat and Temperature. (1 Hour) First Law of Thermodynamics: Differential form, Internal Energy. Equation of state for an adiabatic process, Work Done during Isothermal and Adiabatic Processes. (3 Hours) Second Law of Thermodynamics: Kelvin-Planck and Clausius Statements and their Equivalence. Reversible and Irreversible processes with examples. Heat Engines: Carnot engine & efficiency (no derivation). Refrigeration & coefficient of performance, Applications of Carnot engine in locomotion, Thermodynamic Scale of Temperature and its Equivalence to Perfect Gas Scale. Concept of Entropy, Second Law of Thermodynamics in terms of Entropy (5 Hours) Third Law of Thermodynamics: Statement, Significance and Unattainability of Absolute Zero. (2 Hours)				
Topic Learning Outcomes At the end of the topic, students should be able to:				
SL No	TLO's	BL	CO	PO
i.	Explain the first law of thermodynamics.	L1	1	1-6,11-12
ii.	Give the differential form of the first law of thermodynamics and define what the internal energy is.	L2	1	1-6,11-12
iii.	Obtain an expression for work done in isothermal and adiabatic processes.	L2	1	1-6,11-12
iv.	Give two systems of units of temperature measurement and give their equivalence.	L2	1	1-6,11-12
v.	Describe and Discuss heat engine based on Carnot cycle.	L2	1	1-6,11-12
vi.	Explain how the efficiency of refrigeration is measured?	L2	1	1-6,11-12
vii.	Detail out the application of the Carnot engine to a locomotion system.	L1	1	1-6,11-12
viii.	Define entropy and write an expression for entropy using the second law of thermodynamics.	L2	1	1-6,11-12
ix.	State the third law of thermodynamics and give its significance using the third law of thermodynamics describing why absolute zero temperature is not unattainable.	L2	1	1-6,11-12
x.	High Order Problems.	L3	1	1-6,11-12

Teaching and Learning Methodology	
Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.	
Assessment Techniques	
One minute paper/ Predict-Observe-Explain/ Think-Pair-Share/ Class Test/ Quiz/ Crosswords/ Group Assessment/ Assignment/ Peer-to-Peer Evaluation/Seminar etc	
Suggested Activities (2 Hours)	
Activity No. 1	I feel cold because coldness enter my body. Discuss the statement in day-to-day life. Approximately give examples of (i) open system (ii) closed system and (iii) isolated system Discuss when the temperature of the body is locked until what time you hold the thermometer in contact with a body. Discuss it in contact with laws of thermodynamics. Discuss why when a person works or does exercise, he sweats. Reason it with the laws of thermodynamics.
Activity No. 2	Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. On the specific day, each group has to make a ppt presentation of the following three slides. On the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. (i) The first slide will explain the process of doing the experiment. (ii) In the second slide. Students will show the graph of measurement. (iii) In the third slide, they will list three observations from that study. Activity: Take four different sizes of same metal, preferable of same shape and give one piece to each group. Heat it uniformly on a hot plate. Keep a beaker of water with a thermometer immersed in it. Drop one hot metal into the water and record the temperature with time. Repeat the experiment for the other heated metal pieces of different sizes. (i) Plot a graph for the volume of the metal piece used v/s respective temperature change observed. (ii) Determine the heat capacity and specific heat of the metal used.

	All groups shall also do the following activity:
Activity No. 3	Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. On the specific day, each group has to make a ppt presentation of the following three slides. On the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. (i) The first slide will explain the process of doing the experiment. (ii) In the second slide. Students will show the graph of measurement. (iii) In the third slide, they will list three observations from that study. Activity: Take ice cubes of different size and immerse in water and measure the temperature change with time and repeat the experiment. Graph the observations.

Thermal Physics and Electronics				
Unit – 2				
The Portion to be Covered				
Thermodynamic Potentials: Internal Energy, Enthalpy, Helmholtz Free Energy, Gibb's Free Energy. And their relation. (1 Hour) Maxwell's Thermodynamic Relations: Derivations and applications of Maxwell's Relations (1) First order Phase Transitions with examples, Clausius - Clapeyron Equation (2) Values of Cp-Cv (3) Joule-Thomson Effect and Joule-Thomson coefficient and Derive an equation for Vander Walls gas. Attainment of low temperature by liquefaction of gases and adiabatic demagnetization. (3 Hours) Kinetic Theory of Gases: Distribution of Velocities: Maxwell-Boltzmann Law of Distribution of Velocities in an Ideal Gas: Mean, RMS and Most Probable Speeds. Degrees of Freedom, Law of Equipartition of Energy. Specific heats of Gases. (2 Hours) Radiation: Blackbody radiation, spectral distribution, the concept of energy density and pressure of radiation, Wien's law, Wien's displacement law, Stefan-Boltzmann law, Rayleigh-Jeans law, Planck's law of radiation, derivation of Planck's law of radiation using Einstein's A and B coefficients, derivation of Rayleigh-Jeans law, Wein's displacement law and Stefan's law from Planck's law of radiation, Ultraviolet Radiation catastrophe and. (4 Hours)				
Topic Learning Outcomes				
At the end of the topic, students should be able to:				
SL No	TLO's	BL	CO	PO
i.	State Maxwell relations.	L1	2	1-6, 11-12
ii.	Give examples where Maxwells relations are used.	L1	2	1-6, 11-12

iii.	Explain the phase transition. Which is called as first order phase transition? Give Examples	L2	2	1-6, 11-12
iv.	State Clausius - Clapeyron Equation.	L1	2	1-6, 11-12
v.	Obtain an equation for difference in $C_p - C_v$.	L2	2	1-6, 11-12
vi.	State Joule-Thomson effect and Joule-Thomson coefficient.	L1	2	1-6, 11-12
vii.	Obtain an expression, giving the relation between pressure, volume and temperature for a real gas (Vander Waals gas).	L2	2	1-6, 11-12
viii.	Explain adiabatic demagnetization and how it is used to obtain low temperature by the liquification of gases?	L2	2	1-6, 11-12
ix.	State Maxwell-Boltzmann Law of Distribution of Velocities in Ideal gases.	L1	2	1-6, 11-12
x.	Explain the mean RMS and most probable speeds in ideal gases.	L1	2	1-6, 11-12
xi.	Explain degrees of freedom associated with particles in an ideal gas?	L2	2	1-6, 11-12
xii.	Define the specific heat of a gas.	L1	2	1-6, 11-12
xiii.	Explain black body radiation and its spectral distribution.	L1	2	1-6, 11-12
xiv.	Explain the different laws used to describe different parts of the curves of a spectral distribution of black body radiation.	L2	2	1-6, 11-12
xv.	Define ultraviolet radiation catastrophe? Discuss its importance in the explanation of black body radiation.	L2	2	1-6, 11-12
xvi.	Define Planck's law of radiation and discuss how it could describe the whole black body radiation curve.	L2	2	1-6, 11-12
xvii.	High Order Problems.	L3	2	1-6, 11-12

Teaching and Learning Methodology

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Assessment Techniques

One minute paper/ Predict-Observe-Explain/ Think-Pair-Share/ Class Test/ Quiz/ Crosswords/ Group Assessment/ Assignment/ Peer-to-Peer Evaluation/Seminar etc

Suggested Activities (2 Hours)	
Activity No. 4	(i) Measuring the Solar Constant Materials: Simple flat sided Jar and Thermometer. Activity: Bottle containing water is exposed to solar radiation. The rise in temperature and time taken are noted. Calculate the heat absorbed by water and relate it to the output of the Sun. (ii) Thermo emf Materials: Suitable two dissimilar metal wires, voltage measuring device. Activity: In this experiment student will assemble the thermocouple and study the three effects namely, Seebeck, Peltier, and Thompson. (iii) Inverse square law of radiation Materials: A cardboard with a grid, cardboard with a hole, supporting clips, a ruler, candle. (iv) Activity: Students set the device. They count the lighted squares on the cardboard with the grid by varying the distance. And make necessary measurements and calculations to arrive at the inverse square law of radiation. Ref: Activity Based Physics Thinking Problems in Thermodynamics: Kinetic Theory http://www.physics.umd.edu/perg/abp/think/thermo/kt.htm
Activity No. 5	Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. On the specific day, each group has to make a ppt presentation of the following three slides. On the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. (i) The first slide will explain the process of doing the experiment. (ii) In the second slide. Students will show the graph of measurement. (iii) In the third slide, they will list three observations from that study. Activity: Take two dissimilar metal wires. Spot weld them forming two junctions. Dip one junction in ice and heat the other junction with a burner. Plot a graph of time of heating v/s Thermo EFM generated in the voltmeter.
Activity No. 6	Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. On the specific day, each group has to make a ppt presentation of the following three slides. On the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. (i) The first slide will explain the process of doing the experiment. (ii) In the second slide. Students will show the graph of measurement. (iii) In the third slide, they will list three observations from that study.

	Activity: Make 4 groups and give different-sized balloons to each group. Fit different-sized nozzles into the mouth of the large balloons. Measure the temperature or the EMF generated using a thermocouple placed at the mouth of the nozzle as the pressurised gas is released. Plot a graph of time v/s temperature. Vary the volume of the balloon and repeat the experiment. Plot the graph of volume v/s temperature difference created.
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Thermal Physics and Electronics				
Unit – 3				
The Portion to be Covered				
Semiconductor devices: Review of Intrinsic and Extrinsic semiconductors, p-n junction and its Characteristics and Parameters, Diode approximations, Half-wave rectifier, Full-wave rectifier, Zener diode voltage regulators: Regulator circuit with no load, Loaded Regulator. (5 hours)				
Junction Transistors: Basics of Bipolar Junction Transistors (BJT), BJT operation, Common Base, Common Emitter and Common Collector Characteristics. Field Effect Transistor (FET) and its characteristics. Transistor as an Amplifier- CE Amplifier and Oscillator – Phase Shift Oscillator, Wiens Bridge oscillator. (6 hours)				
Topic Learning Outcomes At the end of the topic, students should be able to:				
SL No	TLO's	BL	CO	PO
i.	Define Semiconductors and Band Gap. Explain on what basis they are classified as intrinsic and extrinsic.	L2	3	1-6, 11-12
ii.	Define PN junction. Explain it's functioning in forward and reverse bias.	L1	3	1-6, 11-12
iii.	Explain the approximation used in a real diode with respect to an ideal PN Junction?	L2	3	1-6, 11-12
iv.	With a schematic diagram, explain half wave and full wave rectifiers.	L1	3	1-6, 11-12
v.	Define a Zener diode and explain how it is different from an ordinary diode using V-I curves?	L2	3	1-6, 11-12
vi.	With the schematic diagram, explain the working of voltage regulators of different types using a Zener diode.	L1	3	1-6, 11-12
vii.	Give the basic concepts used in the instruction of bipolar junction transistor and its operation.	L1	3	1-6, 11-12

viii.	Compare the V-I curve of common base common emitter and common collector BJT curves while explaining their working principles.	L2	3	1-6, 11-12
ix.	Define FET? Give its characteristics.	L1	3	1-6, 11-12
x.	Explain how a transistor can be used as an amplifier and an oscillator using a circuit diagram.	L2	3	1-6, 11-12
xi.	High Order Problems.	L3	3	1-6, 11-12

Teaching and Learning Methodology

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Assessment Techniques

One minute paper/ Predict-Observe-Explain/ Think-Pair-Share/ Class Test/ Quiz/ Crosswords/ Group Assessment/ Assignment/ Peer-to-Peer Evaluation/Seminar etc

Suggested Activities (2 Hours)

Activity No. 7	Wire a regulated DC power supply on a bread board or groove board to give a regulated output voltage of + 5 V; +15 V; Dual power output : ± 5 V; Dual power output : ± 15 V. Use: 3-pin voltage regulators. Components required: 1. Step down transformer- 1 No. (5 V tapping, 100 – 500 mA current rating), BY 127 semiconductor diodes – 4 Nos, Inductor -1, Capacitor - 1, 3 pin 5V regulator-1 Search for circuit diagram in books/net. Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. One the specific day, each group has to make a ppt presentation of the following three slides. One the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. (i) The first slide will explain the process of doing the experiment. (ii) In the second slide. Students will show the graph of measurement. (iii) In the third slide, they will list three observations from that study.
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	Activity: Form 3 groups and tell them to make a DC supply of low current of different voltages like 5V, 10V, and 15V on a breadboard
Activity No. 8	(i) Learn to identify the terminals of different types (packages) of BJTs. (ii) In the case of power transistors, learn how to fix a heat sink for the transistor. (iii) Learn the difference between BJT and FET in its operational characteristics.
Activity No. 9	Note for the teachers for the activity: Make 3-4 groups among students and assign each group the activity of drawing one of the graphs given below. Provide a few days to complete the activity. One the specific day, each group has to make a ppt presentation of the following three slides. One the day of the presentation select a member from each group randomly to make the presentation. Based on the work and presentation, teacher shall assign marks to each group, wherein all members of the group will get equal marks. (i) The first slide will explain the process of doing the experiment. (ii) In the second slide. Students will show the graph of measurement. (iii) In the third slide, they will list three observations from that study. Activity: Take any 3 diode and assign one to each group. Measure its resistance when dipped in ice and heating the ice till it boils. Using this data, plot calibration curve of temperature v/s resistance and also the cooling curve of temperature V/s time for the diode by each group.

Thermal Physics and Electronics				
Unit – 4				
The Portion to be Covered				
Electronics: Integrated Circuits (Analog and Digital), Operational Amplifier, Ideal characteristics of Op-Amp, Inverting and Non-Inverting Configurations. Applications- Voltage Follower, Addition and Subtraction. (4 hours) Digital: Switching and Logic Levels, Digital Waveform. Number Systems: Decimal Number System, Binary Number System, Converting Decimal to Binary, Hexadecimal Number System: Converting Binary to Hexadecimal, Hexadecimal to Binary. (3 hours) Boolean Algebra Theorems: De Morgan's theorem. Digital Circuits: Logic gates, NOT Gate, AND Gate, OR Gate, NAND Gate, NOR Gate, Algebraic Simplification, Implementation of NAND and NOR functions. (4 hours)				
Topic Learning Outcomes At the end of the topic, students should be able to:				
SL No	TLO's	BL	CO	PO

i.	Define op-amps and give the characteristics of an ideal op-amp.	L1	4	1-6, 11-12
ii.	Explains an inverting and non-inverting configuration of typical op-amps, with a schematic diagram.	L2	4	1-6, 11-12
iii.	Explain how op-amps can be used as a voltage follower, with a schematic diagram and with relevant expressions.	L2	4	1-6, 11-12
iv.	Explain how op-amps can be used as a voltage follower, adder and subtractor, with a schematic diagram and with relevant expressions.	L2	4	1-6, 11-12
v.	Give different digital wave forms and explain how one can visualize the switching and logic levels.	L1	5	1-6, 11-12
vi.	Write any four-digit numbers other than zero in the decimal number system and convert that into binary and hexadecimal.	L2	5	1-6, 11-12
vii.	Write any number in a Binary System of 8 digits other than zero and convert it into decimal and hexadecimal.	L2	5	1-6, 11-12
viii.	Write any number in the hexadecimal system of 4 digits other than zero and converted it into a binary and decimal number.	L2	5	1-6, 11-12
ix.	Give simplified diagram for a given Boolean circuit diagram of logic gates, and verify using the De-Morgans theorem.	L2	5	1-6, 11-12
x.	Why are X-NOR gates called Universal Gates?	L2	5	1-6, 11-12
xi.	High Order Problems.	L3	4, 5	1-6, 11-12
Teaching and Learning Methodology				
Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.				
Assessment Techniques				
One minute paper/ Predict-Observe-Explain/ Think-Pair-Share/ Class Test/ Quiz/ Crosswords/ Group Assessment/ Assignment/ Peer-to-Peer Evaluation/Seminar etc				
Suggested Activities (2 Hours)				
Activity No. 10	Learn how to implement logic functions (AND, OR, NOT) using just diodes and resistors.			

	With a circuit diagram show how different types of gates can be built by X-NOR gates.
Activity No. 11	Operational Amplifiers (i) Understand the concept of virtual ground of an OP-AMP. (ii) Learn the different types of op-amps used for different applications. (iii) What is a buffer? Prepare a report on buffers and its application in instrumentation electronics.
Activity No. 12	(i) A man has to take a wolf, a goat, and some cabbage across a river. His rowboat has enough room for the man plus either the wolf or the goat or the cabbage. If he takes the cabbage with him, the wolf will eat the goat. If he takes the wolf, the goat will eat the cabbage. Only when the man is present are the goat and the cabbage safe from their enemies. All the same, the man carries wolf, goat, and cabbage across the river. How? Write the truth table for the above story and implement using gates. (ii) A locker has been rented in the bank. Express the process of opening the locker in terms of digital operation. (iii) A bulb in a staircase has two switches, one switch being at the ground floor and the other one at the first floor. The bulb can be turned ON and also can be turned OFF by and one of the switches irrespective of the state of the other switch. The logic of switching of the bulb resembles.

Textbooks	
Sl No	Title of the Book
1.	Electronic Devices and Circuits, David A. Bell, 2004, PHI, New Delhi
2.	Integrated Electronics, Jacob Millman and CC Halkias
3.	Digital Fundamentals, Floyd, 2001, PHI, New Delhi

References Books	
Sl No	Title of the Book
1.	Heat and Thermodynamics, M.W. Zemansky, Richard Dittman, 1981, McGraw-Hill.
2.	Thermal Physics, S. Garg, R. Bansal and Ghosh, 2nd Edition, 1993, Tata McGraw-Hill
3.	A Treatise on Heat, Meghnad Saha, and B.N.Srivastava, 1958, Indian Press
4.	Modern Thermodynamics with Statistical Mechanics, Carl S. Helrich, 2009, Springer.
5.	Thermodynamics, Kinetic Theory & Statistical Thermodynamics, Sears & Salinger. 1988, Narosa.
6.	An Introduction to Thermal Physics, Daniel V Schroeder, 2020, Oxford University Press

Formative Assessment	
Assessment	Marks
Internal Assessment	10
Activity	10
REU based Group Activity (Conduction, Report, Presentation)	10
Science Communication (Seminar/Poster/ Mini Project etc)	10
Total	40

List of Experiments to be performed in the Laboratory (Minimum EIGHT Experiments have to be carried out)	
1.	Mechanical Equivalent of Heat by Callender and Barne's method
2.	Coefficient of thermal conductivity of Copper by Searle's apparatus
3.	Coefficient of thermal conductivity of a bad conductor by Lee and Charlton's disc method
4.	Determination of Stefan's constant/ Verification of Stefan's law
5.	Determination of Specific heat of a given liquid – Newton's law of cooling
6.	Variation of thermo-emf across two junctions of a thermocouple with temperature
7.	Verification of Clausius –Clapeyron equation and determination of specific enthalpy
8.	V-I Characteristics of Silicon & Germanium PN Junction diodes (FB & RB) V-I Characteristics of Zener Diode and voltage regulator
9.	Characteristics of BJT in Common Emitter Configuration Frequency response of CE Amplifier Frequency response of CC Amplifier (Emitter Follower).
10.	Half Wave and Full Wave Rectifier Without Filter Half Wave and Full Wave Rectifier with Filter
11.	Applications of Operational Amplifier Non-inverting and Inverting op-amp circuits Voltage follower, Adder and Subtractor circuits
12.	Truth table verification of logic gates using TTL 74 series ICs(7400 & 7402). Transfer characteristics of a TTL gate using CRO. Logic Gates; Combinational Circuits; Sequential Circuits
13.	V-I Characteristics FET
14.	Determination of frequency of Phase shift Oscillator
15.	Determination of frequency of Wien Bridge Oscillator

Reference Book for Laboratory Experiments	
Sl No	Title of the Book
1.	Basic Electronics Lab (P242) Manual 2015-16, National Institute of Science Education and Research, Bhubaneswar, 2015.
2.	Suggested Readings: 1. B.L. Worsnop, H.T. Flint, "Advanced Practical Physics for Students", Methuen & Co., Ltd., London, 1962, 9e. 2. S. Panigrahi, B. Mallick, "Engineering Practical Physics", Cengage Learning India Pvt. Ltd., 2015, 1e.

Open Electives for I Semester

OE-1: ENERGY SOURCES

Duration: 3 hrs/week

Max Marks: 60

		No. of lectures
Unit-I	Non-Renewable energy sources	
	Chapter-1: Introduction	
	Energy concept-sources in general, its significance & necessity. Classification of energy sources: Primary and Secondary energy, Commercial and Non-commercial energy, Renewable and Non-renewable energy, Conventional and Non-conventional energy, Based on Origin-Examples and limitations. Importance of Non-commercial energy resources.	04
	Chapter-2: Conventional energy sources	
	Fossil fuels & Nuclear energy- production & extraction, usage rate and limitations. Impact on environment and their issues & challenges. Overview of Indian & world energy scenario with latest statistics-consumption & necessity. Need of eco-friendly & green energy & their related technology.	09
	Total	13
Unit-II	Renewable energy sources	
	Chapter-1: Introduction:	
	Need of renewable energy, non-conventional energy sources. An overview of developments in Offshore Wind Energy, Tidal Energy, Wave energy systems, Ocean Thermal Energy Conversion, solar energy, biomass, biochemical conversion, biogas generation, geothermal energy tidal energy, Hydroelectricity.	05
	Chapter 2 : Solar energy:	
	Solar Energy-Key features, its importance, Merits & demerits of solar energy, Applications of solar energy. Solar water heater, flat plate collector, solar distillation, solar cooker, solar green houses, solar cell -brief discussion of each. Need and characteristics of photovoltaic (PV) systems, PV models and equivalent circuits, and sun tracking systems.	08
	Total	13
Unit-III	Chapter-3: Wind and Tidal Energy harvesting:	
	Fundamentals of Wind energy, Wind Turbines and different electrical machines in wind turbines, Power electronic interfaces, and grid interconnection topologies. Ocean Energy Potential against Wind and Solar, Wave Characteristics and Statistics, Wave Energy Devices. Tide characteristics and Statistics, Tide Energy Technologies, Ocean Thermal Energy.	07

	Chapter-4 : Geothermal and hydro energy	
	Geothermal Resources, Geothermal Technologies.	02
	Hydropower resources, hydropower technologies, environmental impact of hydro power sources.	03
	Carbon captured technologies, cell, batteries, power consumption	01
	Total	13
	Reference Books: 1. Non-conventional energy sources - G.D Rai - Khanna Publishers, New Delhi 2. Solar energy - M P Agarwal - S Chand and Co. Ltd. 3. Solar energy - Suhas P Sukhative Tata McGraw - Hill Publishing Company Ltd. 4. Godfrey Boyle, "Renewable Energy, Power for a sustainable future", 2004, Oxford University Press, in association with The Open University. 5. Dr. P Jayakumar, Solar Energy: Resource Assessment Handbook, 2009 6. J.Balfour, M.Shaw and S. Jarosek, Photovoltaics, Lawrence J Goodrich (USA). 7. http://en.wikipedia.org/wiki/Renewable_energy	

OE-2: Climate Science

Duration: 3 hrs /week

Max Marks: 60

Chapter 1:	Atmosphere Atmospheric Science (Meteorology) as a multidisciplinary science. Physical and dynamic meteorology, Some terminology, difference between weather and climate, weather and climate variables, composition of the present atmosphere: fixed and variable gases, volume mixing ratio (VMR), sources and sinks of gases in the atmosphere. Green house gases. Structure (layers) of the atmosphere. Temperature variation in the atmosphere, temperature lapse rate, mass, pressure and density variation in the atmosphere. Distribution of winds.	(13 hours)
Chapter 2:	Climate Science Overview of meteorological observations, measurement of: temperature, humidity, wind speed and direction and pressure. Surface weather stations, upper air observational network, satellite observation. Overview of clouds and precipitation, aerosol size and concentration, nucleation, droplet growth and condensation (qualitative description). Cloud seeding, lightning and discharge. Formation of trade winds, cyclones. Modelling of the atmosphere: General principles, Overview of General Circulation Models (GCM) for weather forecasting and prediction. Limitations of the models. R and D institutions in India and abroad dedicated to climate Science, NARL, IITM, CSIR Centre for Mathematical Modeling and Computer Simulation, and many more	(13 hours)
Chapter 3:	Global Climate Change Green house effect and global warming, Enhancement in concentration of carbon dioxide and other green house gases in the atmosphere, Conventional and non-conventional energy sources and their usage. EL Nino/LA Nino Southern oscillations. Causes for global warming: Deforestation, fossil fuel burning, industrialization. Manifestations of global warming: Sea level rise, melting of glaciers, variation in monsoon patterns, increase in frequency and intensity of cyclones, hurricanes, tornadoes. Geo-engineering as a tool to mitigate global warming? Schemes of Geo-engineering.	(13 hours)
	References: 1. Basics of Atmospheric Science – A Chndrashekar, PHI Learning Private Ltd. New Delhi, 2010. 2. Fundamentals of Atmospheric Modelling- Mark Z Jacobson, Cambridge University Press, 2000.	

Open Electives for II Semester

OE-3: Astronomy

Duration: 3 hrs/week

Max Marks: 60

Content		Hrs
Unit – 1 -History and Introduction		
Chapter 1	Ancient Astronomy Greek Observations, Sumerian Observations, Mayan Observations, Arabic Observations ,Chinese Observations	2
Chapter 2	Indian Astronomy Vedic Astronomy, Ancient Astronomy – Aryabhata, Varahamihira, Bhaskara, Astronomy in Indian Scriptures, Precession of the Equinox, Celebrations of Equinox	2
Chapter 3	Medieval & Modern Astronomy Invention of Telescopes, Models of the Solar System & Universe, Observations by Tycho Brahe, Kepler, Galileo, Herschel and Other, Modern Astronomy	2
Chapter 4	Optical tools for Astronomy - Pin Hole, Binoculars, Telescopes & Imaging.	1
Chapter 5	Mathematical Methods of Observations Angular Measurement, Trigonometric functions, Stellar Parallax	1
Chapter 6	Observational Terminologies Cardinal Directions, Azimuth, Altitude, Measurements using Compass and Hand. Equatorial Co-ordinates, Light years, Magnitude, Colors etc.	2
Unit – 2: Unit 2: Observations of the Solar System		
Chapter 7.	The Sun Ecliptic and the Orientation of the Earth, Seasons - Solstices and Equinox, Observations of the Sun from Earth during seasons. Eclipses, Zero-shadow day, Sunspots	1
Chapter 8	The Moon Earth-Moon system – Phases, Lunar Eclipses, Ecliptic and Lunar Orbital Plane – Nodes, Lunar Month, Full Moon Names	1
Chapter 9.	Inner Planets: Mercury & Venus Observational History, Observational Windows, Appearance, Apparitions, Elongations, Superior Conjunctions, Inferior Conjunctions, Transits.	2
Chapter 10	Outer Planets Outer Planets: Mars, Jupiter & Saturn Observational History. Observational Windows, Appearance, Frequency of Oppositions, Oppositions, Conjunctions, Moons Eclipses. Galilean Moons, Saturn's Rings	2

Unit III Major Astronomy Observations

Chapter 11	March to June Prominent Stars and Constellations Visible during this period, Methods of Spotting.	2
Chapter 12	June to September Prominent Stars and Constellations Visible during this period, Methods of Spotting.	2
Chapter 13	September to December Prominent Stars and Constellations Visible during this period, Methods of Spotting.	2
Chapter 14	December to March Prominent Stars and Constellations Visible during this period, Methods of Spotting.	2

Reference Books:

1. The Stargazer's Guide - How to Read Our Night Sky by Emily Winterburn
2. A guide to the Night Sky – Beginner's handbook by P.N. Shankar
3. The Complete Idiot's guide to Astronomy by Christopher De Pree and Alan Axelrod

Text Books

1. P. N. SHANKAR A GUIDE TO THE NIGHT SKY
<https://www.arvindguptatoys.com/arvindgupta/nightskyshankar.pdf>
2. Biman Basu, Joy of Star Watching, National Book Trust of India 2013

References Books

Christopher De Pree :The Complete Idiot's Guide to Astronomy, Penguin USA, 2008
Emily Winterburn, The Stargazer's Guide: How to Read Our Night Sky, Constable and Robinson, 2008

OE-4: Medical Physics

Duration: 3 hrs /week

Max Marks: 60

Unit I:	Human Anatomy and Physiology Overview of human anatomy - cells, cell structure, type of cells and their functions, tissues, organs, and their functions. Different systems in the humanbody, their structure and function, physiological properties of the circulatory system, digestive system, respiratory system, reproductive system, excretory system, endocrine system and nervous system	(13 hours)
Unit II:	Physics of Medical Diagnostics Principle of production of X-rays. Use of X-rays in medical diagnosis, X-ray imaging systems. Computed Tomography (CT): principle and generation of CT. Magnetic Resonance Imaging (MRI): basic principle and image characteristics. Ultrasound Imaging: Interaction of sound waves with body tissues, production of ultrasound, transducers, acoustic coupling, image formation, modes of image display and color Doppler.	(13 hours)
Unit III:	Physics of Radiotherapy Clinical aspects of radiation therapy: Biological basis of radiotherapy, radiation sources, radiation dose, time dose fractionation. External beam radiation therapy, radiation therapy modalities, production of radioisotopes, use of radioisotopes in therapy, particle and ion beam radiotherapy. Brachytherapy - principle of brachytherapy and classification of brachytherapy techniques.	(13 hours)

Text Books

1. C. H. Best and N. B. Taylor. A Text in Applied Physiology. Williams and Wilkins Company, Baltimore, 1999.
2. C. K. Warrick. Anatomy and Physiology for Radiographers. Oxford University Press, 2001.
3. Jerrold T. Bushberg. The Essential Physics for Medical Imaging (2ndEdition). Lippincott Williams & Wilkins, 2002.
4. Jean A. Pope. Medical Physics: Imaging. Heinemann Publishers, 2012.
5. Faiz M. Khan and Roger A. Potish. Treatment Planning in Radiation Oncology. Williams and Wilkins, USA, 2003.
6. D. Baltas. The physics of modern brachytherapy for oncology. Taylor and Francis, 2007.

Reference Books

1. J. R. Brobek. Physiological Basis of Medical Practice. Williams and Wilkins, London, 1995.
2. Edward Alcamo, Barbara Krumhardt. Barron's Anatomy and Physiology the Easy Way. Barron's Educational Series, 2004.

3. Lippincott, Anatomy and Physiology. Lippincott Williams & Wilkins, 2002.
4. W. E. Arnould Taylor. A textbook of anatomy and physiology, Nelson Thornes, 1998.
5. G. S. Pant. Advances in Diagnostic Medical Physics. Himalaya Publishing House, 2006.
6. Sabbahaga, Diagnostic Ultrasound applied to OBG. Maryland, 1980.
7. Faiz M Khan. The Physics of Radiation Therapy (3rd edition). Lippincott Williams & Wilkins, USA, 2003.
8. Jatinder R. Palta and T. Rockwell Mackie. Intensity Modulation Radiation Therapy. Medical Physics publishing, Madison, Wisconsin, 2003.
9. AAPM Report No. 72. Basic Applications of Multileaf collimators, AAPM, USA, 2001.
10. AAPM Report No. 91. Management of Respiratory motion in radiation oncology, 2006.
11. CA Joslin, A. Flynn, E. J. Hall. Principles and Practice of Brachytherapy. Arnold publications, 2001.
12. Peter Hoskin, Catherine Coyle. Radiotherapy in Practice. Oxford University Press, 2011.
13. W. R. Handee. Medical Radiation Physics. Year Book Medical Publishers Inc., London, 2003.
14. Donald T. Graham, Paul J. Cloke. Principles of Radiological Physics. Churchill Livingstone, 2003.
15. Thomas S. Curry. Christensen's Physics of Diagnostic Radiology (4th Edition). Lippincott Williams & Wilkins, 1990.
16. Madison. MRI – Perry Sprawls – Medical Physics Publishing. Wisconsin

Open Electives for III Semester

OE-5: OPTICAL INSTRUMENTS

Duration: 3 hrs./week

Max Marks: 60

Unit 1.	Basics of Optics Scope of optics, optical path, laws of reflection and refraction as per Fermat's principle, magnifying glass, Lenses (thick and thin), convex and concave lenses, Lens makers formulae for double concave and convex lenses, lens equation. Focal and nodal points, focal length, image formation, combination of lenses, dispersion of light: Newton's experiment, angular dispersion and dispersion power. Dispersion without deviation. (Expressions need not be derived, but have to be discussed qualitatively).	13
Unit 2.	Camera and microscopes Human eye (constitution and working), Photographic camera (principle, construction and working), construction, working and utilities of Simple microscopes, Compound microscope, Electron microscopes, Binocular microscopes Self study Experimental determination of magnifying power of a microscope. (Construction part can be discussed through block diagrams)	13
Unit 3.	Telescopes and Spectrometer Construction, working and utilities of Astronomical telescopes Terrestrial telescopes Reflecting telescopes, Construction, working and utilities of Eyepieces or Oculars (Huygen, Ramsden's, Gauss) Spectrometer - Construction, working and utilities, measurement of refractive index. Self study Telescopes used at different observatories in and outside India.	13
	Activities: Find position and size of the image in a magnifying glass and magnification. Observe rain bows and understand optics. Create a rainbow. Find out what makes a camera to be of good quality. Observe the dispersion of light through prism. Make a simple telescope using magnifying glass and lenses. Learn principle of refraction using prisms. Check bending of light in different substances and find out what matters here. Learn about different telescopes used to see galaxies and their ranges. Many more activities can be tried to learn optics by going through you tubes and webistes such as https://spark.iop.org, http://www.yenka.com, https://publiclab.org etc.	

OE-6: Sports Science

Duration: 3 hrs./week

Max Marks: 60

Content (Use maths of 10 th Std only – Only qualitative discussion)		Hrs
Unit - 1		
Chapter No. 1	Measurement: Physical quantities. Standards and Units. International system of Units. Standards of time, length and mass. Precision and significant figures.	04
Chapter No. 2	Newton's laws of motion: Newton's first law. Force, mass. Newton's second law. Newton's third law. Mass and weight. Applications of Newton's laws.	03
Chapter No. 3	Projectile motion: Shooting a falling target. Physics behind Shooting, Javelin throw and Discus throw.	03
Topics for self study (If any)	https://www.real-world-physics-problems.com/physics-of-sports.html	
Unit - 2		
Chapter No. 4.	Conservation laws: Conservation of linear momentum, collisions – elastic and inelastic. Angular momentum. (Physics behind Carom, Billiards, Racing)	04
Chapter No. 5.	Centre of mass: Physics behind Cycling, rock climbing, Skating,	02
Chapter No. 6.	Gravitation: Origin, Newton's law of gravitation. Archimedes's principle, Buoyancy (Physics behind swimming)	04
Topics for self study (If any)	Archimedes' Principle: Made EASY Physics in You tube	
Unit - 3		
Chapter No.7	Food and Nutrition: Proteins, Vitamins, Fat, Blood pressure. Problems due to the deficiency of vitamins.	04
Chapter No. 8	Energy: Different forms of Energy, Conservation of mass-energy.	03
Chapter No . 9	Physical exercises: Walking, Jogging and Running, Weight management.	03
Topics for self study (If any)	10 Best Exercises for Everyone – Healthline	
Suggested Activities		
Activity No. 1	Identify the methods of measurement of time, length and mass from ancient time and build models for them.	02
	Reference : History of measurement - Wikipedia https://en.wikipedia.org › wiki › History_of_measurem	
Activity No. 2	Identify Physics principles behind various Sports activities.	01

	https://www.real-world-physics-problems.com/physics-of-sports.html	
Activity No. 3	List the difficulties experienced in Gymnastics, Cycling and weight lifting.	02
Activity No. 4	List the difficulties experienced in swimming.	01
Activity No. 3	List the difficulties experienced in Gymnastics, Cycling and weight lifting.	02
Activity No. 4	List the difficulties experienced in swimming.	01
Activity No. 5	Learn breathing exercises.	02
	Reference : 1) Simple Breathing Exercise for Beginners Swami Ramdev 2) https://www.yogajournal.com	
Activity No.6	Write an essay on Physical health v/s Mental health or conduct a debate on Physical health v/s Mental health.	01

Text Books

Sl No	Title of the Book	Authors Name	Publisher	Year of Publication
1	Physics for Entertainment	Yakov Perelman	Createspace Independent Pub.	
2	Physics Everywhere	Yakov Perelman	Prodinnova	2014
3	Mechanics for Entertainment	Yakov Perelman	Prodinnova	2014
4	Handbook of Food and Nutrition	M.Swaminathan	Bangalore Press 2012	2012
5	Food Science	B. Srilakshmi	New Age International Pub	2015

References Books

Sl No	Title of the Book	Authors Name	Publisher	Year of Publication
1	Physics	Resnick, Halliday and Krane, Vol 1	Wiley Student Edition.	
2	For the love of Physics	Walter Lewin	Taxmann Publications Private Limited	2012
3	An Introduction to the Physics of Sports	VassiliosMcInnesS pathopoulos	CreateSpace Independent Publishing Platform	2013

Internet resources

<https://www.topendsports.com/biomechanics/physics.htm>
<https://www.real-world-physics-problems.com/physics-of-sports.html>
<https://www.healthline.com/>
<https://www.mayoclinic.org/>
<https://www.who.int/news-room/>

OE-7: NANOTECHNOLOGY

Duration: 3 hrs./week

Max Marks: 60

Unit 1:	Introduction to nanomaterials Length scales in physics, Nanostructures: 1D, 2D and 3D nanostructures (nano dots, thin films, nanowires, nanorods), Band structure and density of states of materials at nanoscale, Size Effects in nano systems, Quantum confinement: Applications of Schrodinger equation Infinite potential well, potential step, potential box, quantum confinement of carriers in 3D, 2D, 1D nanostructures and its consequences.	(13hours)
Unit 2:	Synthesis and Characterization of nanostructure materials Top down and Bottom up approach, Photolithography. Ball milling. Gas phase condensation. Vacuum deposition. Physical vapor deposition (PVD): Thermal evaporation, E-beam evaporation, Pulsed Laser deposition. Chemical vapor deposition (CVD). Sol-Gel. Electrodeposition. Spray pyrolysis. Hydrothermal synthesis. Preparation through colloidal methods. MBE growth of quantum dots. X-Ray Diffraction. Optical Microscopy. Scanning Electron Microscopy. Transmission Electron Microscopy. Atomic Force Microscopy. Scanning Tunneling Microscopy.	(13 hours)
Unit 3:	Properties and applications of nanomaterials Coulomb interaction in nanostructures. Concept of dielectric constant for nanostructures and charging of nanostructure. Quasi-particles and excitons. Excitons in direct and indirect bandgap semiconductor nanocrystals. Quantitative treatment of quasiparticles and excitons, charging effects. Radiative processes: General formalization-absorption, emission and luminescence. Optical properties of heterostructures and nanostructures. Applications of nanoparticles, quantum dots, nanowires and thin films for photonic devices (LED, solar cells). Nanomaterial Devices: Quantum dots heterostructure lasers, optical switching and optical data storage. Magnetic quantum well; magnetic dots - magnetic data storage.	(13 hours)
References Books: ➤ C.P. Poole, Jr. Frank J. Owens, Introduction to Nanotechnology (Wiley India Pvt. Ltd.). ➤ S.K. Kulkarni, Nanotechnology: Principles & Practices (Capital Publishing Company) ➤ K.K. Chattopadhyay and A. N. Banerjee, Introduction to Nanoscience and Technology (PHI Learning Private Limited). ➤ Richard Booker, Earl Boysen, Nanotechnology (John Wiley and Sons). ➤ M. Hosokawa, K. Nogi, M. Naita, T. Yokoyama, Nanoparticle Technology Handbook (Elsevier, 2007). ➤ Introduction to Nanoelectronics, V.V. Mitin, V.A. Kochelap and M.A. Strosio, 2011, Cambridge University Press. ➤ Bharat Bhushan, Springer Handbook of Nanotechnology (Springer-Verlag, Berlin, 2004).		

Student Activities:

1. Synthesis of metal nanoparticles by chemical route.
2. Synthesis of semiconductor nanoparticles.
3. XRD pattern of nanomaterials and estimation of particle size.
4. To study the effect of size on color of nanomaterials.
5. Growth of quantum dots by thermal evaporation.
6. Prepare a disc of ceramic of a compound using ball milling, pressing and sintering, and study its XRD.
7. Fabricate a thin film of nanoparticles by spin coating (or chemical route) and study transmittance spectra in UV-Visible region.
8. Prepare a thin film capacitor and measure capacitance as a function of temperature or frequency.
9. Visit to nearby research labs to study the working of XRD, SEM, UV-Visible Spectrophotometer instruments
10. Visit to nearby research labs for project work and interaction with scientists at IISC, JNC SR, Universities etc.

OE- 8: ELECTRICAL INSTRUMENTS

Duration: 3 hrs./week

Max Marks: 60

Content		Hrs
Unit - 1		
Chapter No. 1	Voltage and current sources, Kirchoff's current and voltage laws, loop and nodal analysis of simple circuits with dc excitation. Ammeters,voltmeters: (DC/AC)	03
Chapter No. 2	Representation of sinusoidal waveforms, peak and rms values, power factor. Analysis of single-phase series and parallel R-L-C ac circuits. Three-phase balanced circuits, voltage and current relations in star and delta connections. Wattmeters: Induction type, single phase and three phase wattmeter, Energy meters: AC. Induction type single phase and three phase energy meter	05
Chapter No. 3	Instrument Transformers: Potential and current transformers, ratio and phase angle errors, phasor diagram, methods of minimizing errors; testing and applications.	05
Topics for self study (If any)	Types of switches and Circuits, Safety precautions and rules in handling electrical appliances, Electric shock, first aid for electrical shocks, Fuses, MCB, ELCB and Relays, Filament lamp, Tube light, CFL and LED	
Suggested Activities		
Activity No. 1	Identify variety of electrical switches and note down their applications/utility. Reference: Weblink/Youtube/Book	

Activity No. 2	Identify the hazards involved in handling electrical circuits and instruments, make a list of safety precautions as well as first aid for electrical shocks. Reference : Weblink/Youtube/Book	
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Unit - 2		
Chapter No. 4.	Galvanometers: General principle and performance equations of D'ArsonvalGalvanometers, Vibration Galva nometer and Ballistic Galvanometer.	03
Chapter No. 5.	Potentiometers: DCPotentiometer, Crompton potentio meter, construction, standardization, application. AC Potentio meter, Drysdalepolar potentio meter; standardization, application.	03
Chapter No. 6.	DC/AC Bridges: General equations for bridge balance, measurement of self inductance by Maxwell's bridge (with variable inductance & variable capacitance), Hay's bridge, Owen's bridge, measurement of capacitance by Schearing bridge, errors, Wagner's earthing device, Kelvin's double bridge.	07
Topics for self study (If any)	Importance of grounding and <u>Earthing</u> , Methods for <u>Earthing</u> ,	

	Suggested Activities	
Activity No. 3	Make a study of importance of grounding in electrical circuits.	
	Reference : Weblink/Youtube/Book	
Activity No. 4	Prepare a detailed account of various methods of earthing and their utility/applications	
	Reference : Weblink/Youtube/Book	
Unit - 3		
Chapter No.7	Transducer: Strain Gauges, Thermistors, Thermocouples, Linear Variable Differential Transformer (LVDT), Capacitive Transducers, Piezo-Electric transducers, Optical Transducer, Hall Effect Transducer	06
Chapter No. 8	CRO: Block diagram, Sweep generation, vertical amplifiers, use of CRO in measurement of frequency, phase, Amplitude and rise time of a pulse. Digital Multi-meter: Block diagram, principle of operation	03
Chapter No. 9	Basics of lead acid batteries, Lithium Ion Battery, Battery storage capacity, Coulomb efficiency, Numerical of high and low charging rates, Battery sizing.	04
Topics for self study (If any)	Fuses, MCB, ELCB and Relays, Filament lamp, Tube light, CFL and LED	
	Suggested Activities	
Activity No. 5	Prepare a document on evolution of incandescent bulbs to the present day LED lights	
	Reference : Weblink/Youtube/Book	

Activity No.6	Make a comparative study of Fuses, MCB, ELCB and Relays highlighting their use and applications	
	Reference : Weblink/Youtube/Book	

Text

Books

AK.Sawhney, A Course in Elec.&Electronics Measurements&Instrumentation, Dhanpatrai & Co. 1978
A.D. Helfrick & W.D. Cooper, Modern Electronic Instrumentation and Measurement Techniques PHI, 2016

References Books

1. D C Kulshreshtha, Basic Electrical Engineering, Mc Graw Hill Publications, 2019
2. David G Alciatore and Michel B Hstand, Introduction to Mechatronics and Measurement Systems, 3rd, Tata McGraw Hill Education Private Limited, New Delhi., 2005
3. Vincent Del Toro, Electrical Engineering Fundamentals Prentice Hall India 2009

B.Sc in Physics V Semester

Program Name	BSc in Physics		Semester	V
Course Title	Classical Mechanics and Quantum Mechanics- I (Theory)			
Course Code	PHY C9-T		No. of Credits	04
Contact Hours	60 Hours		Duration of SEA/Exam	02 Hours
Formative Marks	Assessment	40	Summative Assessment Marks	60

Course Pre-requisite(s):

Course Outcomes (COs): After the successful completion of the course, the student will be able to

- Identify the failure of classical physics at the microscopic level.
- Find the relationship between the normalization of a wave function and the ability to correctly calculate expectation values or probability densities.
- Explain the minimum uncertainty of measuring both observables on any quantum state.
- Describe the time-dependent and time-independent Schrödinger equation for simple potentials like for instance one-dimensional potential well and Harmonic oscillator.
- Apply Hermitian operators, their eigenvalues and eigenvectors to find various commutation and uncertainty relations.

Contents	60 Hrs
Introduction to Newtonian Mechanics: Frames of references, Newton's laws of motion, inertial and non-inertial frames. Mechanics of a particle, Conservation of linear momentum, Angular momentum and torque, conservation of angular momentum, work done by a force, conservative force and conservative energy. Lagrangian formulation: Constraints, Holonomic constraints, non-holonomic constraints, Scleronomic and Rheonomic constraints. Generalized coordinates, degrees of freedom, Principle of virtual work, D'Alembert's principle, Lagrange equations. Newton's equation of motion from Lagrange equations, simple pendulum, Atwood's machine and linear harmonic oscillator. 12 Hours Activities: 03 Hours	15
Variational principle: Hamilton's principle, Deduction of Hamilton's principle, Lagrange's equation of motion from Hamilton's principle, Hamilton's principle for non-holonomic systems. Hamiltonian Mechanics: The Hamiltonian of a system, Hamilton's equations of motion, Hamilton's equations from variational principle, Integrals of Hamilton's equations, energy integrals, Canonical Transformations, Poisson Brackets, fundamental properties and equations of motion in Poisson Brackets. 12Hours Activities: 03 Hours	15

Introduction to Quantum Mechanics Brief discussion on failure of classical physics to explain black body radiation, Photoelectric effect, Compton effect, stability of atoms and spectra of atoms. Compton scattering: Expression for Compton shift (With derivation). Matter waves: de Broglie hypothesis of matter waves, Electron microscope, Wave description of particles by wave packets, Group and Phase velocities and relation between them, Experimental evidence for matter waves: Davisson- Germer experiment, G.P Thomson's experiment and its significance. Heisenberg uncertainty principle: Elementary proof of Heisenberg's relation between momentum and position, energy and time, angular momentum and angular position, illustration of uncertainty principle by Gamma ray microscope thought experiment. Consequences of the uncertainty relations: Diffraction of electrons at a single slit, why electron cannot exist in nucleus? Two-slit experiment with photons and electrons. Linear superposition principle as a consequence. 12 Hours Activities: 03 Hours	15
Foundation of Quantum Mechanics Probabilistic interpretation of the wave function - normalization and orthogonality of wave functions, Admissibility conditions on a wave function, Schrödinger equation: equation of motion of matter waves - Schrodinger wave equation for a free particle in one and three-dimension, time-dependent and time-independent wave equations, Probability current density, equation of continuity and its physical significance, Postulates of Quantum mechanics: States as normalized wavefunctions. Dynamical variables as linear Hermitian operators (position, momentum, angular momentum, and energy as examples). Expectation values of operators and their time evolution. Ehrenfest theorem (no derivation), Commutator brackets- Simultaneous Eigen functions, Commutator bracket using position, momentum and angular momentum operators. Particle in a one-dimensional infinite potential well (derivation), degeneracy in three-dimensional case, particle in a finite potential well (qualitative), Transmission across a potential barrier, the tunnel effect (qualitative), scanning tunnelling microscope, One-dimensional simple harmonic oscillator (qualitative) - concept of zero - point energy. 12 Hours Activities: 03 Hours	15

Pedagogy: Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.

Formative Assessment for Theory		
Assessment type	Occasion/	Marks
	C1	20
	C2	20
Total		40 Marks
Formative Assessment as per UNIVERSITY guidelines are compulsory		

References	
1	Classical Mechanics, H.Goldstein, C.P. Poole, J.L. Safko, 3rd Edn. 2002, Pearson Education.
2	Classical Mechanics: An introduction, Dieter Strauch, 2009, Springer
3	Classical Mechanics, G. Aruldas, 2008, Prentice-Hall of India Private limited, New Delhi.
4	Classical Mechanics, Takwale and Puranik-1989, Tata McGraw Hill, new Delhi
5	Concepts of Modern Physics, Arthur Beiser, McGraw-Hill, 2009.
6	Physics for Scientists and Engineers with Modern Physics, Serway and Jewett, 9th edition, Cengage Learning, 2014.
7	Quantum Physics, Berkeley Physics Course Vol. 4. E.H. Wichman, Tata McGraw-Hill Co., 2008.
8	Six Ideas that Shaped Physics: Particle Behave like Waves, Thomas A. Moore, McGraw Hill, 2003.
9	P M Mathews and K Venkatesan, A Textbook of Quantum Mechanics, Tata McGraw Hill publication, ISBN: 9780070146174.
10	Ajoy Ghatak, S. Lokanathan, Quantum Mechanics: Theory and Applications, Springer Publication, ISBN 978-1-4020-2130-5.
11	Modern Physics; R.Murugesan & K.Sivaprasath; S. Chand Publishing.
12	G Aruldas, Quantum Mechanics, Phi Learning Private Ltd., ISBN: 97881203363.
13	Gupta, Kumar & Sharma, Quantum Mechanics, Jai Prakash Nath Publications.
14	Physics for Degree Students B.Sc., Third Year, C.L.Arora and P.S.Hemne, 1st edition, S.Chand & Company Pvt. Ltd., 2014.

Course Title	Classical Mechanics and Quantum Mechanics- I (Practical)	Practical Credits	02
Course Code	PHY C10-P	Contact Hours	04 Hours
Formative Assessment	25 Marks	Summative Assessment	25 Marks
Practical Content			
Lab experiments: (at least 4 experiments from 1-8 and 4 experiments from 9-18) 1) To determine 'g', the acceleration due to gravity, at a given place, from the $L - T^2$ graph, for a simple pendulum. 2) Studying the effect of mass of the bob on the time period of the simple pendulum. [Hint: With the same experimental set-up, take a few bobs of different materials (different masses) but of same size. Keep the length of the pendulum same for each case. Starting from a small angular displacement of about 10° find out, in each case, the time period of the pendulum, using bobs of different masses. Does the time period depend on the mass of the pendulum bob? If yes, then see the order in which the change occurs. If not, then do you see an additional reason to use the pendulum as a time measuring device.] 3) Studying the effect of amplitude of oscillation on the time period of the simple pendulum. [Hint: With the same experimental set-up, keep the mass of the bob and length of the pendulum fixed. For measuring the angular amplitude, make a large protractor on the cardboard and have a scale marked on an arc from 0° to 90° in units of 5°. Fix it on the edge of a table by two drawing pins such that its 0°-line coincides with the suspension thread of the pendulum at rest. Start the pendulum oscillating with a very large angular amplitude (say 70°) and find the time period T of the pendulum. Change the amplitude of oscillation of the bob in small steps of 5° or 10° and determine the time period in each case till the amplitude becomes small (say 5°). Draw a graph between angular amplitude and T. How does the time period of the pendulum change with the amplitude of oscillation? How much does the value of T for $A = 10^\circ$ differ from that for $A = 50^\circ$ from the graph you have drawn? Find at what amplitude of oscillation, the time period begins to vary? Determine the limit for the pendulum when it ceases to be a simple pendulum.] 4) Determine the acceleration of gravity is to use an Atwood's machine/Fly Wheel. 5) Study the conservation of energy and momentum using projectile motion. 6) Verification of the Principle of Conservation of Linear Momentum 7) A code in Python-Scilab to plot and analyze the trajectory of projectile motion 8) Determination of acceleration due to gravity by Stoke's method 9) Determination of Planck constant and work function of the material of the cathode using Photo-electric cell. 10) To study the spectral characteristics of a photo-voltaic cell (Solar cell). 11) Determination of electron charge 'e' by Millikan's Oil drop experiment. 12) To study the characteristics of solar cell. 13) To find the value of e/m for an electron by Thomson's method using bar magnets. 14) To determine the value of e/m for an electron by magnetron method. 15) To study the tunnelling in Tunnel Diode using I-V characteristics. 16) Determination of quantum efficiency of Photodiode. 17) A code in Python-Scilab to find the first seven eigen states and eigen functions of Linear Harmonic Oscillator by solving the Schrödinger equation. 18) A code in Python-Scilab to plot and analyse the wavefunctions for particle in an infinite potential well.			

Lab experiments: (at least 4 experiments from 1-8 and 4 experiments from 9-18)

19) To determine 'g', the acceleration due to gravity, at a given place, from the $L - T^2$ graph, for a simple pendulum.

20) Studying the effect of mass of the bob on the time period of the simple pendulum.

[Hint: With the same experimental set-up, take a few bobs of different materials (different masses) but of same size. Keep the length of the pendulum same for each case. Starting from a small angular displacement of about 10° find out, in each case, the time period of the pendulum, using bobs of different masses. Does the time period depend on the mass of the pendulum bob? If yes, then see the order in which the change occurs. If not, then do you see an additional reason to use the pendulum as a time measuring device.

21) Studying the effect of amplitude of oscillation on the time period of the simple pendulum.

[Hint: With the same experimental set-up, keep the mass of the bob and length of the pendulum fixed. For measuring the angular amplitude, make a large protractor on the cardboard and have a scale marked on an arc from 0° to 90° in units of 5° . Fix it on the edge of a table by two drawing pins such that its 0° -line coincides with the suspension thread of the pendulum at rest. Start the pendulum oscillating with a very large angular amplitude (say 70°) and find the time period T of the pendulum. Change the amplitude of oscillation of the bob in small steps of 5° or 10° and determine the time period in each case till the amplitude becomes small (say 5°). Draw a graph between angular amplitude and T . How does the time period of the pendulum change with the amplitude of oscillation? How much does the value of T for $A = 10^\circ$ differ from that for $A = 50^\circ$ from the graph you have drawn? Find at what amplitude of oscillation, the time period begins to vary? Determine the limit for the pendulum when it ceases to be a simple pendulum.]

22) Determine the acceleration of gravity is to use an Atwood's machine/Fly Wheel.

23) Study the conservation of energy and momentum using projectile motion.

24) Verification of the Principle of Conservation of Linear Momentum

25) A code in Python-Scilab to plot and analyze the trajectory of projectile motion

26) Determination of acceleration due to gravity by Stoke's method

27) Determination of Planck constant and work function of the material of the cathode using Photo-electric cell.

28) To study the spectral characteristics of a photo-voltaic cell (Solar cell).

29) Determination of electron charge 'e' by Millikan's Oil drop experiment.

30) To study the characteristics of solar cell.

31) To find the value of e/m for an electron by Thomson's method using bar magnets.

32) To determine the value of e/m for an electron by magnetron method.

33) To study the tunnelling in Tunnel Diode using I-V characteristics.

34) Determination of quantum efficiency of Photodiode.

35) A code in Python-Scilab to find the first seven eigen states and eigen functions of Linear Harmonic Oscillator by solving the Schrödinger equation.

36) A code in Python-Scilab to plot and analyse the wavefunctions for particle in an infinite potential well.

Pedagogy: Demonstration/Experiential Learning / Self Directed Learning etc.

Formative Assessment for Practical		
Assessment type	Occasion/	Marks
	C1	10
	C2	15
Total		25 Marks

Formative Assessment as per UNIVERSITY guidelines are compulsory

References	
1	B.Sc Practical Physics by C.L Arora.
2	B.Sc Practical Physics by Harnam Singh and P.S Hemne.
3	Practical Physics by G.S Squires.
4	Scilab Manual for CC-XI: Quantum Mechanics & Applications (32221501) by Dr Neetu Agrawal, Daulat Ram College of Delhi.
5	Scilab Textbook Companion for Quantum Mechanics by M. C. Jain.
6	Computational Quantum Mechanics using Scilab, BIT Mesra.
7	Advanced Practical Physics for Students by Worsnop B L and Flint H T.

Curriculum

Program Name	BSc in Physics	Semester	V
Course Title	Elements of Atomic, Molecular & Laser Physics (Theory)		
Course Code	PHY C11-T	No. of Credits	04
Contact Hours	60 Hours	Duration of SEA/Exam	02 Hours
Formative Assessment Marks	40	Summative Assessment Marks	60

Course Pre-requisite (s): PUC Science Knowledge	
Course Outcomes (COs): After the completion of the course, the student will be able to • Describe atomic properties using basic atomic models. • Interpret atomic spectra of elements using vector atom model. • Interpret molecular spectra of compounds using basics of molecular physics. • Explain laser systems and their applications in various fields.	
Contents	60 Hours

Basic Atomic models Thomson's atomic model; Rutherford atomic model – Model, Theory of alpha particle scattering, Rutherford scattering formula; Bohr atomic model – postulates, Derivation of expression for radius, total energy of electron; Origin of the spectral lines; Spectral series of hydrogen atom; Effect of nuclear motion on atomic spectra - derivation; Ritz combination principle; Correspondence principle; Critical potentials – critical potential, excitation potential and ionisation potential; Atomic excitation and its types, Franck-Hertz experiment; Sommerfeld's atomic model – model, Derivation of condition for allowed elliptical orbits. 12 Hours Activities: 03 Hours 1. Students to estimate radii of orbits and energies of electron in case of hydrogen atom in different orbits and plot the graph of radii / energy versus principal quantum number 'n'. Analyze the nature of the graph and draw the inferences. 2. Students to search critical, excitation and ionisation potentials of different elements and plot the graph of critical /excitation / ionisation potentials versus atomic number/mass number/neutron number of element. Analyze the nature of the graph and draw the inferences.	15
Vector atomic model and optical spectra Vector atom model – model fundamentals, spatial quantisation, spinning electron; Quantum numbers associated with vector atomic model; Coupling schemes – L-S and j-j schemes; Pauli's exclusion principle; Magnetic dipole moment due to orbital motion of electron – derivation; Magnetic dipole moment due to spin motion of electron; Lande g-factor and its calculation for different states; Stern-Gerlach experiment – Experimental arrangement and Principle; Fine structure of spectral lines with examples; Spin-orbit coupling/Spin-Orbit Interaction – qualitative; Optical spectra – spectral terms, spectral notations, selection rules, intensity rules; Fine structure of the sodium D-line; Zeeman effect: Types, Experimental study and classical theory of normal Zeeman effect, Zeeman shift expression (no derivation), examples; Stark	15

effect: Experimental study, Types and examples.	12Hours	
Activities: 1. Students to couple a p-state and s-state electron via L-S and j-j coupling schemes for a system with two electrons and construct vector diagrams for each resultant. Analyze the coupling results and draw the inferences. 2. Students to estimate magnetic dipole moment due to orbital motion of electron for different states $^2P_{1/2}$, $^2P_{3/2}$, $^2P_{5/2}$, $^2P_{7/2}$, $^2P_{9/2}$ and $^2P_{11/2}$ and plot the graph of dipole moment versus total orbital angular momentum “J”. Analyze the nature of the graph and draw the inferences.	03 Hours	
Molecular Physics Types of molecules based on their moment of inertia; Types of molecular motions and energies; Born-Oppenheimer approximation; Origin of molecular spectra; Nature of molecular spectra; Theory of rigid rotator – energy levels and spectrum, Qualitative discussion on Non-rigid rotator and centrifugal distortion; Theory of vibrating molecule as a simple harmonic oscillator – energy levels and spectrum; Electronic spectra of molecules – fluorescence and phosphorescence; Raman effect – Stoke’s and anti-Stoke’s lines, characteristics of Raman spectra, classical and quantum approaches, Experimental study of Raman effect; Applications of Raman effect. Activities:	12 Hours 03 Hours	15
Laser Physics Ordinary light versus laser light; Characteristics of laser light; Interaction of radiation with matter - Induced absorption, spontaneous emission and stimulated emission with mention of rate equations; Einstein’s A and B coefficients – Derivation of relation between Einstein’s coefficients and radiation energy density; Possibility of amplification of light; Population inversion; Methods of pumping; Metastable states; Requisites of laser – energy source, active medium and laser cavity; Difference between Three level and four level lasers with examples; Types of lasers with examples; Construction and Working principle of Ruby Laser and He-Ne Laser; Application of lasers (qualitative) in science & research, isotope separation, communication, fusion, medicine, industry, war and space. Activities:	12 Hours 03 Hours	15

Pedagogy: Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/

Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.

Formative Assessment for Theory		
Assessment type	Occasion/	Marks
	C1	20
	C2	20
Total		40 Marks
<i>Formative Assessment as per UNIVERSITY guidelines are compulsory</i>		

References	
1	Modern Physics, R. Murugesan, Kiruthiga Sivaprakash, Revised Edition, 2009, S. Chand & Company Ltd.
2	Atomic & Molecular spectra: Laser, Raj Kumar, Revised Edition, 2008, Kedar Nath Ram Nath Publishers, Meerut.
3	Atomic Physics, S.N. Ghoshal, Revised Edition, 2013, S. Chand & Company Ltd.
4	Concepts of Atomic Physics, S.P. Kuila, First Edition, 2018, New Central Book Agency (P) Ltd.
5	Concepts of Modern Physics, Arthur Beiser, Seventh Edition, 2015, Shobhit Mahajan, S. Rai Choudhury, 2002, McGraw-Hill.
6	Fundamentals of Molecular Spectroscopy, C.N. Banwell and E.M. McCash, Fourth Edition, 2008, Tata McGraw-Hill Publishers.
7	Elements of Spectroscopy – Atomic, Molecular and Laser Physics, Gupta, Kumar and Sharma, 2016, Pragati Publications.

Course Title	Elements of Atomic, Molecular & Laser Physics (Practical)			Practical Credits	02
Course Code	PHY C12-P			Contact Hours	04 Hours
Formative Assessment	25 Marks		Summative Assessment		25 Marks
Practical Content					

NOTE: Students have to perform at-least EIGHT Experiments from the list below

LIST OF EXPERIMENTS

1. To determine Planck's constant using Photocell.
2. To determine Planck's constant using LED.
3. To determine wavelength of spectral lines of mercury source using spectrometer.
4. To determine the value of Rydberg's constant using diffraction grating and hydrogen discharge tube.
5. To determine the wavelength of H-alpha emission line of Hydrogen atom.
6. To determine fine structure constant using fine structure separation of sodium D-lines using a plane diffraction grating.
7. To determine the ionization potential of Mercury/Xenon.
8. To determine the absorption lines in the rotational spectrum of Iodine vapour.
9. To determine the force constant and vibrational constant for the iodine molecule from its absorption spectrum.
10. To determine the wavelength of laser using diffraction by single slit/double slits.
11. To determine wavelength of He-Ne laser using plane diffraction grating.
12. To determine angular spread of He-Ne laser using plane diffraction grating.
13. Study of Raman scattering by CCl_4 using laser and spectrometer/CDS.
14. To determine the diameter of the given wire by LASER diffraction.
15. Analysis of Stellar Spectra.
16. Analysis of Band Spectra.

Pedagogy: Demonstration/Experiential Learning / Self Directed Learning etc.

Formative Assessment for Practical		
Assessment type	Occasion/	Marks
	C1	10
	C2	15
Total		25 Marks
<i>Formative Assessment as per UNIVERSITY guidelines are compulsory</i>		

References	
1	Practical Physics, D.C. Tayal, First Millennium Edition, 2000, Himalaya Publishing House.
2	B.Sc. Practical Physics, C.L. Arora, Revised Edition, 2007, S. Chand & Comp.Ltd.
3	An Advanced Course in Practical Physics, D. Chatopadhyaya, P.C. Rakshith, B. Saha, Revised Edition, 2002, New Central Book Agency Pvt. Ltd.
4	Physics through experiments, B. Saraf, 2013, Vikas Publications.

B.Sc in Physics VI Semester

Curriculum

Program Name	BSc in Physics		Semester	VI
Course Title	Elements of Condensed Matter & Nuclear Physics(Theory)			
Course Code	PHY C14 - T		No. of Credits	4
Contact Hours	60 Hours		Duration of SEA/Exam	2 Hours
Formative Marks	Assessment	40	Summative Assessment Marks	60

4

Course Pre-requisite(s):	
Course Outcomes (COs): After the successful completion of the course, the student will be able to: • Explain the basic properties of nucleus and get the idea of its inner information. • Understand the concepts of binding energy and binding energy per nucleon v/s mass number graph. • Describe the processes of alpha, beta and gamma decays based on well-established theories. • Explain the basic aspects of interaction of gamma radiation with matter by photoelectric effect, Compton scattering and pair production. • Explain the different nuclear radiation detectors such as ionization chamber, Geiger-Mueller counter etc. • Explain the basic concept of scintillation detectors, photo-multiplier tube and semiconductor detectors.	
Contents	60 Hours
Crystal systems and X-rays: Crystal structure: Space Lattice, Lattice translational vectors, Basis of crystal structure, Types of unit cells, primitive, non-primitive cells.. Seven crystal system, Coordination numbers, Miller Indices, Expression for inter planner spacing. X Rays: Production and properties of X rays, Coolidge tube, Continuous and characteristic X-ray spectra; Moseley's law. X-Ray diffraction, Scattering of X-rays, Bragg's law. Crystal diffraction: Bragg's X-ray spectrometer- powder diffraction method, Intensity vs 2θ plot (qualitative). Free electron theory of metals: Classical free electron model (Drude-Lorentz model), expression for electrical and thermal conductivity, Weidman-Franz law, Failure of classical free electron theory; Quantum free electron theory, Fermi level and Fermi energy, Fermi-Dirac distribution function (expression for probability distribution $F(E)$, statement only); Fermi Dirac distribution at $T=0$ and $E < E_f$, at $T \neq 0$ and $E > E_f$, $F(E)$ vs E plot at $T = 0$ and $T \neq 0$. Density of states for free electrons (statement only, no derivation). Qualitative discussion of lattice vibration and concept of Phonons. Specific heats of solids: Classical theory, Einstein's and Debye's theory of specific heats. Hall Effect in metals. 12 HOURS ACTIVITIES: 03 HOURS	15
Magnetic Properties of Matter, Dielectrics and Superconductivity Magnetic Properties of Matter Review of basic formulae: Magnetic intensity, magnetic induction, permeability, magnetic susceptibility, magnetization (M), Classification of Dia, Para, and ferro magnetic materials;	15

Langevin Classical Theory of dia – and Paramagnetism. Curie’s law, Ferromagnetism and Ferromagnetic Domains (qualitative). Discussion of B-H Curve. Hysteresis and Energy Loss, Hard and Soft magnetic materials Dielectrics: Static dielectric constant, polarizability (electronic, ionic and orientation), calculation of Lorentz field (derivation), Clausius-Mosotti equation (derivation), dielectric loss. Piezo electric effect, cause, examples and applications. Superconductivity: Definition, Experimental results – Zero resistivity and Critical temperature– The critical magnetic field – Meissner effect, Type I and type II superconductors. 12 Hours ACTIVITIES: 03 Hours	
General Properties of Nuclei: Constituents of nucleus and their intrinsic properties, quantitative facts about mass, radii, charge density (matter density), binding energy, main features of binding energy versus mass number curve, angular momentum, parity, magnetic moment, electric moments Radioactivity decay: Radioactivity: definition of radioactivity, half-life, mean life, radioactivity equilibrium (a) Alpha decay: basics of α-decay processes, theory of α emission (brief), Gamow factor, Geiger-Nuttall law. (b) β-decay: energy kinematics for β-decay, positron emission, electron capture, neutrino hypothesis. (c) Gamma decay: Gamma rays’ emission & kinematics, internal conversion (Definition). 12 Hours ACTIVITIES: 03 Hours	15
Interaction of Nuclear Radiation with matter: Gamma ray interaction through matter, photoelectric effect, Compton scattering, pair production, Energy loss due to ionization (quantitative description of Bethe Block formula), energy loss of electrons, introduction of Cerenkov radiation Detector for Nuclear Radiations: Gas detectors: estimation of electric field, mobility of particle, for ionization chamber and GM Counter. Basic principle of Scintillation Detectors and construction of photo-multiplier tube (PMT). Semiconductor Detectors (Si and Ge) for charge particle and photon detection (concept of charge carrier and mobility) qualitative only, Accelerators: Cyclotrons and Synchrotrons. 12 Hours ACTIVITIES: 03 Hours	15
Suggested Activities:	
1) Students to construct seven crystal systems with bamboo sticks and rubber bands. Use foam ball as atoms and study the BCC and FCC systems. 2) Students to search the characteristic X ray wavelength of different atoms/elements and plot characteristic wavelength vs atomic number and analyse the result and draw the inference. 3) Magnetic field lines are invisible. Students to trace the magnetic field lines using bar magnet and needle compass. https://nationalmaglab.org/magnet-academy/try-this-at-home/drawing-magnetic-field-lines/ , 4) Using vegetable oil and iron fillings students to make ferrofluids and see how it behaves in the presence of magnetic field. https://nationalmaglab.org/magnet-academy/try-this-at-home/making-ferrofluids/ 1) Study the decay scheme of selected alpha, beta & gamma radioactive sources with the help of standard nuclear data book. 2) Calculate binding energy of some selected light, medium and heavy nuclei. Plot the graph of binding energy versus mass number A 3) Study the decay scheme of standard alpha, beta and gamma sources using nuclear data book. 4) Make the list of alpha emitters from Uranium series and Thorium series. Search the kinetic energy of alpha particle emitted by these alpha emitters. Collect the required data such as half life or decay constant. Verify Geiger-Nuttall in each series. 5) Study the Z dependence of photoelectric effect cross section. 6) Study the Z dependence of common cross section for selected gamma energies and selected elements through theoretical calculation. 7) List the materials and their properties which are used for photocathode of PMT.	

8) Study any two types of PMT and their advantages and disadvantages.	
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Pedagogy: Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.

Formative Assessment for Theory	
Assessment Occasion/ type	Marks
C1	20
C2	20
Total	40 Marks
<i>Formative Assessment as per UNIVERSITY guidelines are compulsory</i>	

References
1. Solid State Physics-R. K. Puri and V.K. Babber., S.Chand publications,1st Edition(2004). 2. Fundamentals of Solid State Physics-B.S.Saxena,P.N. Saxena,Pragati prakashan Meerut(2017). 3. Introductory nuclear Physics by Kenneth S. Krane (Wiley India Pvt. Ltd., 2008). 4. Nuclear Physics, Irving Kaplan, Narosa Publishing House 1. Introduction to solid State Physics, Charles Kittel, VII edition, (1996) 5. Solid State Physics- A J Dekker, MacMillan India Ltd, (2000) 6. Essential of crystallography, M A Wahab, Narosa Publications (2009) 7. Solid State Physics-S O Pillai-New Age Int. Publishers (2001). 8. Concepts of nuclear physics by Bernard L. Cohen. (Tata McGraw Hill, 1998). 9. Introduction to the physics of nuclei & particles, R.A. Dunlap. (Thomson Asia, 2004). 10. Introduction to High Energy Physics, D.H. Perkins, Cambridge Univ. Press 11. Basic ideas and concepts in Nuclear Physics - An Introductory Approach by K. Heyde (Institute of Physics (IOP) Publishing, 2004). 12. Radiation detection and measurement, G.F. Knoll (John Wiley & Sons, 2000). 13. Physics and Engineering of Radiation Detection, Syed Naeem Ahmed (Academic Press, Elsevier, 2007).

Course Title	Elements of Condensed Matter & Nuclear Physics (Practical)	Practical Credits	02
Course Code	PHY C15 – P	Contact Hours	04 Hours
Formative Assessment	25 Marks	Summative Assessment	25 Marks
Practical Content			

(At least 4 experiments from CMP and 4 experiments from NP are to be performed)

CONDENSED MATTER PHYSICS(CMP)

1. Hall Effect in semiconductor: determination of mobility, hall coefficient.
2. Energy gap of semiconductor (diode/transistor).
3. Temperature coefficient of resistance of a Thermistor.
4. Fermi Energy of Copper.
5. Analysis of X-ray diffraction spectra and calculation of lattice parameter.
6. Specific Heat of Solid by Electrical Method
7. Determination of Dielectric Constant of polar liquid.
8. Determination of dipole moment of organic liquid
9. B-H Curve Using CRO.
10. Determination of particle size from XRD pattern using Debye-Scherrer formula.
11. Measurement of susceptibility of paramagnetic solution (Quinck's Tube Method).
12. Measurement of susceptibility of paramagnetic solid (Gouy's Method)
13. Determination of particle size from XRD pattern using Williamson Hall Plot.

NUCLEAR PHYSICS(NP)

1. Study the characteristics of Geiger-Müller Tube. Determine the threshold voltage, plateau region and operating voltage.
2. Study of inverse square law of gamma rays using GM tube.
3. Determination of range of electrons in aluminium using GM Counter.
4. Study the absorption of beta particles in aluminium foils using GM counter. Determine mass attenuation coefficient of Aluminium foils.
5. Study the absorption of beta particles in thin copper foils using G M counter and determine mass attenuation coefficient.
6. Study the attenuation of gamma rays in lead foils using Cs-137 source and G M counter. Calculate mass attenuation coefficient of Lead for Gamma.
7. Determine the end point energy of Tl-204 source by studying the absorption of beta particles in aluminium foils.
Study the attenuation of absorption of gamma rays in polymeric materials using Cs-137 source and G M counter.

Pedagogy: Demonstration/Experiential Learning / Self Directed Learning etc.

Formative Assessment for Practical		
Assessment type	Occasion/	Marks
	C1	10
	C2	15
Total		25 Marks
<i>Formative Assessment as per UNIVERSITY guidelines are compulsory</i>		
References		
1	IGNOU : Practical Physics Manual	
2	Saraf : Experiment in Physics, Vikas Publications	
3	S.P. Singh : Advanced Practical Physics	
4	Melissons : Experiments in Modern Physics	
5	Misra and Misra, Physics Lab. Manual, South Asian publishers, (2000)	
6	Gupta and Kumar, Practical physics, Pragati prakashan, (1976)	

Curriculum

Program Name	BSc in Physics	Semester	VI
Course Title	Electronic Instrumentation & Sensors (Theory)		
Course Code:	PHY C16 - T	No. of Credits	04
Contact Hours	60 Hours	Duration of SEA/Exam	2 Hours
Formative Marks	Assessment 40	Summative Assessment Marks	60

Course Pre-requisite(s):

Course Outcomes (COs): After the successful completion of the course, the student will be able to:

- Identify different types of tests and measuring instruments used in practice and understand their basic working principles.
- Get hands on training in wiring a circuit, soldering, making a measurement using an electronic circuit used in instrumentation.
- Have an understanding of the basic electronic components viz., resistors, capacitors, inductors, discrete and integrated circuits, colour codes, values and pin diagram, their practical use.
- Understanding of the measurement of voltage, current, resistance value, identification of the terminals of a transistor and ICs.
- Identify and understand the different types of transducers and sensors used in robust and hand-held instruments.
- Understand and give a mathematical treatment of the working of rectifiers, filter, data converters and different types of transducers.
- Connect the concepts learnt in the course to their practical use in daily life.
- Develop basic hands-on skills in the usage of oscilloscopes, multimeters, rectifiers, amplifiers, oscillators and high voltage probes, generators and digital meters.
- Servicing of simple faults of domestic appliances: Iron box, immersion heater, fan, hot plate, battery charger, emergency lamp and the like.

Contents	60Hours
Power supply AC power and its characteristics, Single phase and three phase, Need for DC power supply and its characteristics, line voltage and frequency, Rectifier bridge, Filters: Capacitor and inductor filters, L-section and π-section filters, ripple factor, electronic voltage regulators, stabilization factor, voltage regulation using ICs. Basic electrical measuring instruments Cathode ray oscilloscope- Block diagram, basic principle, electron beam, CRT features, signal display. Basic elements of digital storage oscilloscopes. Basic DC voltmeter for measuring potential difference, Extending Voltmeter range, AC voltmeter using rectifiers, Basic DC ammeter, requirement of a shunt, Extending of ammeter ranges. <i>Topics for self-study:</i> <i>Average value and RMS value of current, Ripple factor, Average AC input power and DC output power, efficiency of a DC power supply. Multirange voltmeter and ammeter.</i> 12 Hours ACTIVITIES: Activities 03 Hours	15

Design and wire your own DC regulated power supply. Power output: 5 V, 10 V, ± 5 V. Components required: A step down transformer, semiconductor diodes (BY126/127), Inductor, Capacitor, Zener diode or 3-pin voltage regulator or IC. Measure the ripple factor and efficiency at each stage. Tabulate the result. 1. Extend the range of measurement of voltage of a voltmeter (analog or digital) using external component and circuitry. Design your own circuit and report. 2. Measure the characteristics of the signal waveform using a CRO and function generator. Tabulate the frequency and time period. Learn the function of Trigger input in an CRO. 3. Learn to use a Storage Oscilloscope for measuring the characteristics of a repetitive input signal. Convince yourself how signal averaging using Storage CRO improves S/N ratio.	
Wave form generators and Filters Basic principle of standard AF signal generator: Fixed frequency and variable frequency, AF sine and square wave generator, basic Wein-bridge network and oscillator configuration, Triangular and saw tooth wave generators, circuitry and waveforms. Passive and active filters. Fundamental theorem of filters, Proof of the theorem by considering a symmetrical T-network. Types of filters, Circuitry and Cut-off frequency and frequency response of Passive (RC) and Active (op-amp based) filters: Low pass, high pass and band pass. 12 Hours ACTIVITIES: 03 Hours Activities 1. Measure the amplitude and frequency of the different waveforms and tabulate the results. Required instruments: A 10 MHz oscilloscope, Function generators (sine wave and square wave). 2. Explore where signal filtering network is used in real life. Visit a nearby telephone exchange and discuss with the Engineers and technicians. Prepare a report. 3. Explore op-amp which works from a single supply biasing voltage (+15V). Construct an inverting/non-inverting amplifier powered by a single supply voltage instead of dual or bipolar supply voltage. 4. Op-amp is a linear (analog) IC. Can it be used to function as logic gates? Explore, construct and implement AND, OR NAND and NOR gate functions using op-amps. Verify the truth table. Hint: LM3900 op-amp may be used. The status of the output may be checked by LED.	15
Data Conversion and display Digital to Analog (D/A) and Analog to Digital (A/D) converters – A/D converter with pre-amplification and filtering. D/A converter - Variable resistor network, Ladder type (R-2R) D/A converter, Op-amp based D/A converter. Digital display systems and Indicators- Classification of displays, Light Emitting Diodes (LED) and Liquid Crystal Display (LCD) – Structure and working. Data Transmission systems – Advantages and disadvantages of digital transmission over analog transmission, Pulse amplitude modulation (PAM), Pulse time modulation (PTM) and Pulse width modulation (PWM)- General principles. Principle of Phase Sensitive Detection (PSD). <i>Topic for self-study: Lock-in amplifier and its application, phase locked loop.</i> 12 Hours ACTIVITIES: 03 Hours Activities 1. Explore where modulation and demodulation technique is employed in real life. Visit a Radio broadcasting station. (Aakashvani or Private). Prepare a report on different AM and FM stations. 2. Explore and find out the difference between a standard op-amp and an instrumentation op-amp. Compare the two and prepare a report.	15
Transducers and sensors Definition and types of transducers. Basic characteristics of an electrical transducer, factors governing the selection of a transducer, Resistive transducer-potentiometer, Strain gauge and types (general description), Resistance thermometer-platinum resistance thermometer.	15

Thermistor. Inductive Transducer-general principles, Linear Variable Differential Transducer (LDVT)- principle and construction, Capacitive Transducer, Piezo-electric transducer, Photoelectric transducer, Photovoltaic cell, photo diode and phototransistor – principle and working. ACTIVITIES: Activities - Construct your own thermocouple for the measurement of temperature with copper and constantan wires. Use the thermocouple and a Digital multimeter (DMM). Record the emf (voltage induced) by maintaining one of the junctions at a constant temperature (say at 0° C, melting ice) and another junction at variable temperature bath. Tabulate the voltages induced and temperatures read out using standard chart (Chart can be downloaded from the internet). - Observe a solar water heater. Some solar water heaters are fitted with an anode rod (alloy of aluminium). Study why it is required. Describe the principle behind solar water heater.	12 Hours 03 Hours
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Pedagogy: Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/ Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.

Formative Assessment for Theory	
Assessment Occasion/ type	Marks
C1	20
C2	20
Total	40 Marks
<i>Formative Assessment as per UNIVERSITY guidelines are compulsory</i>	

References
- Physics for Degree students (Third Year) – C.L. Arora and P.S. Hemne, S, Chand and Co. Pvt. Ltd. 2014 (For Unit-1, Power supplies) - Electronic Instrumentation, 3rd Edition, H.S. Kalsi, McGraw Hill Education India Pvt. Ltd. 2011 (For rest of the syllabus) - Instrumentation – Devices and Systems (2nd Edition)– C.S. Rangan, G.R. Sarma, V.S.V. Mani, Tata McGraw Hill Education Pvt. Ltd. (Especially for circuitry and analysis of signal generators and filters)

Course Title	Electronic Instrumentation & Sensors (Practical)	Practical Credits	02
Course Code	PHY C17 – P	Contact Hours	04 Hours
Formative Assessment	25 Marks	Summative Assessment	25 Marks

Practical Content
List of experiments (At least 8 experiments to be performed) 1. Bridge rectifier with and without filter 2. Phase measurement in LCR circuit using CRO 3. Study of Zener diode as a voltage regulator. 4. RC low pass and high pass filters. 5. Calibration of a low range voltmeter using a potentiometer 6. Calibration of an ammeter using a potentiometer 7. Study of Wien bridge oscillator 8. Study the frequency response of a first order op-amp low pass filter 9. Study the frequency response of a first order op-amp high pass filter 10. Study of LDR Characteristics. 11. Study the characteristics of pn-junction of a solar cell and determine its efficiency. 12. Study the illumination intensity of a solar cell using a standard photo detector (e.g., lux meter). 13. Determine the coupling coefficient of a piezo-electric crystal. 14. Study the amplitude modulation using a transistor. 15. Performance analysis of A/D and D/A converter using resistor ladder network and op-amp.

Pedagogy: Lecture/ PPT/ Videos/ Animations/ Role Plays/ Think-Pair-Share/ Predict-Observe-Explain/ Demonstration/ Concept mapping/ Case Studies examples/ Tutorial/ Activity/ Flipped Classroom/ Jigsaw/Field based Learning/ Project Based Learning/ Mini Projects/ Hobby Projects/ Forum Theatre/ Dance/ Problem Based Learning/ Game Based Learning/ Group Discussion/ Collaborative Learning/ Experiential Learning / Self Directed Learning etc.

Formative Assessment for Practical	
Assessment Occasion/ type	Marks
C1	10
C2	15
Total	25 Marks
Formative Assessment as per University guidelines are compulsory	

References
1. Advanced Practical Physics for students, B. L. Flint and H.T. Worsnop, 1971, Asia Publishing House. 2. B.Sc. Practical Physics, C.L. Arora (Revised Edition), S. Chand and Co. Ltd. 2007 3. Practical Physics, D.C. Tayal, First Millennium Edition, Himalaya Publishing House, 2000

Employability and skill development

The whole syllabus is prepared with a focus on employability.

Skill development achieved: Fundamental understanding of the working of test and measuring instruments. Operating and using them for measurements. Servicing of laboratory equipment for simple cable faults, loose contacts and discontinuity.

Job opportunities: Lab Assistant/Scientific Assistant in hospitals, R and D institutions, educational institutions.

Question Paper Pattern

DSC Papers

I to VI SEMESTER - THEORY

Duration: $2\frac{1}{2}$ Hrs

Maximum marks: 60

PART- A

I Answer the following questions:

4 x 10 = 40

EIGHT questions have to be set and FOUR questions to be answered.
(TWO questions must be given from each unit.)

1. a
b

OR

2. a
b

3. a
b

OR

4. a
b

5. a
b

OR

6. a
b

7. a
b

OR

8. a
b

PART - B

II Answer any THREE questions:

3 x 4 = 12

(Problems/Application Oriented Questions)
FOUR questions have to be set THREE questions to be answered.
(ONE question from each unit has to be given.)

PART - C

III Answer any FOUR questions:

4 x 2 = 08

SIX questions have to be set and FOUR questions to be answered.
(ONE question is a must from each unit.)

Question Paper Pattern

Open Elective Papers

I to VI SEMESTER

THEORY

Duration: 2Hrs

Maximum marks: 60

PART- A

I Answer any SIX questions:

6 x 10 = 60

NINE questions has to be set and SIX questions to be answered
(THREE questions from each unit have to be set. Each question may include sub questions of both theory and numerical.)

Practical Examination-Scheme of Valuation

I to IV Semesters

Duration: 3 Hours

Max. Marks: 25

Content	Marks
Diagram/Circuit diagram	02
Formula with units, explanation of terms	02
Circuit connections and handling	03
Tabular column with units	02
Observations/method of taking reading/Tabulation	04
Calculation	05
Result with accuracy and unit	02
Viva	05
Total marks for practical examination	25

JSS College for Women (Autonomous), Saraswathipuram, Mysuru-09
Department of Physics
PANEL OF EXAMINERS

SL.NO	NAME OF EXAMINER	NAME OF THE COLLEGE	CONTACT NO
1.	Sridevi Dyvanagoudar	JSS College, Nanjanagud	
2.	Vinay kumar.L	JSS College ooty road, Mysore	
3.	Karthik	JSS College ooty road, Mysore	
4.	Chaithra	JSS College ooty road, Mysore	
5.	Shwetha U S	JSS College ooty road, Mysore	
6.	Mallikarjunswamy	JSS College, chamarajanagar	
7.	Lakshmi S	JSS College, chamarajanagar	
8.	C. Nagesh babu	Yuvaraja's College, Mysore	
9.	Dr. B.C. Manjunath	Yuvaraja's College, Mysore	
10.	T.Sadashiviah	Yuvaraja's College, Mysore	
11.	Dr.Somashekarappa	Yuvaraja's College, Mysore	
12.	Sushma	Yuvaraja's College, Mysore	
13.	Sunitha Rani	Yuvaraja's College, Mysore	
14.	Sukhanth	Yuvaraja's College, Mysore	
15.	Dr S R Kumaraswamy	Maharani's Science College, Mysore	
16.	Dr. Manjunath M.V.	Maharani's Science College, Mysore	
17.	Dr. G.B.Thippeswamy	Maharani's Science College,Mysore	
18.	Dr.Krishnamohan	Maharani's Science College,Mysore	
19.	Nagaraju	Maharani's Science College,Mysore	
20.	Rashmi P E	Maharani's Science College,Mysore	
21.	Reema	Maharani's Science College,Mysore	
22.	Rashmi	Maharani's Science College,Mysore	
23.	Vindu vahini	Maharani's Science College,Mysore	
24.	Dr.Jayasheelan	Maharani's Science College,Mysore	
25.	Kavitha	Maharani's Science College,Mysore	
26.	Bharathi N	SDM college, Mysore	
27.	Dr.Gunasheelan	St Philomenas college Mysore	
28.	Dr.Nagaraj Urs	St Philomenas college Mysore	
29.	Dr.R Manjunatha	Mahajanas college Mysore	

30.	Dr.Poornima	Mahajanas college Mysore	
31.	Dr.Roopa	Govt First grade college Gundlupet	
32.	Dr.Mahadevaprasad T N	GFGC, Gundlupet	
33.	Sophia	Teresian college, Mysore	
34.	Dr.Annie Mathew	Teresian college, Mysore	
35.	Dr.Srinivs P	GFGC, Kollegal	
36.	Lakshmi N M	GFGC, Kollegal	
37.	Sri. S Shreekanth	NIE College, Mysore	
38.	Sri G.Shivakumaraswamy	JSS College ooty road, Mysore	
39.	Umesh V.	JSSCW,Saraswathipuram, Mysore	
40.	Yashwanth D.B.	JSSCW,Saraswathipuram, Mysore	
41.	Meghana R	JSSCW,Saraswathipuram, Mysore	
42.	Chandan N.	GFGC, Chamaraja Nagar	
43.	Manjunath	GFGC, Chamaraja Nagar	
44.	Galabe Satishbabu	GFGC, Mandya	
45.	Krupashree P	Yuvaraja's College	
46.	Girish	Maharani's Science College	
47.	Nanda Kumar	Maharani's Science College	

Chairman, BOE is authorized to approve additional members if required.

BBA/BCom-Curriculum and Credit Framework for Bachelor of Business Administration/Bachelor of Commerce.

Sem.	Discipline Specific – Core (DSC), Elective (DSE) Courses (Credits) (L+T+P)	Minor/ Multidisciplinary/ Open Elective (OE) Courses (Credits) (L+T+P)	Ability Enhancement Courses (AEC)(Credits) (L+T+P) (Languages)	Skills Enhancement Courses (SEC) (Credits) (L+T+P)/ Value Added Courses (Credits) (L+T+P) (common for all UG Programs)/ Summer Internship.		Total Credits
I	BA/Com DSC-C1(4), C2(4), C3(4).	OE-1 (3)	L1-1(3), L2-1(3) (4 hrs each)	SEC-1: Digital Fluency (2) (1+0+2)/ Env. Studies (3)	Health, Wellness & Yoga (2) (1+0+2)	25/26
II	BA/Com DSC-C4(4), C5(4), C6(4).	OE-2 (3)	L1-2(3), L2-2(3) (4 hrs each)	Env. Studies (3)/ SEC-1: Digital Fluency (2)(1+0+2)	Sports/NCC/NSS/R&R(S&G)/ Cultural (2) (0+0+4)/ SEC (2)	26/25
Students exiting the programme after securing 46 credits will be awarded UG Certificate in Disciplines provided they secure 4 credits in work based vocational courses during summer term or internship/Apprenticeship in addition to 6 credits from skill-based courses earned during the first year.						
III	BA/Com DSC-C7(4), C8(4), C9(4).	OE-3 (3)/ India and Indian Constitution (3)	L1-3(3), L2-3(3) (4 hrs. each)	SEC-2:AI/Cyber Security/Financial Edu. & Inv. Aw. (2) (1+0+2)	Sports/NCC/NSS/R&R(S&G)/ Cultural (2) (0+0+4) /SEC (2)	25
IV	BA/Com DSC-C10(4), C11 (4), C12 (4).	India and Indian Constitution (3)/ OE-3(3)	L1-4(3), L2-4(3) (4 hrs. each)	SEC-3: Financial Edu. &Inv. Aw. /AI /Cyber Security (2) (1+0+2)	Sports/NCC/NSS/R&R(S&G)/ Cultural (2) (0+0+4)/ SEC (2)	25
Students exiting the programme after securing 92 credits will be awarded UG Diploma in Disciplines or inter-disciplines provided they secure additional 4 credits in skill based vocational courses offered during the first- or second- year summer term.						
V	BA/Com DSC-C13(4), C14(4), C15(4).	BA/BCom DSE-E1(3), E2(3), Vocational-1 (3)		SEC-4: Employability Skills (3) (2+0+2)		24
VI	BA/Com DSC-C16(4), C17(4), C18(4).	BA/Com DSE-E3(3), E4(3), Vocational-2 (3)		Internship (3)		24



Participates Handbook on
“Employability Skills”

by

Karnataka State Higher Education Council

and

Karnataka Skill Development Corporation

Preface

Aptitude skills are very important for students to prepare for any competitive examinations. There is a need for the graduates who are aspiring to prepare for competitive examinations of guidance right from their graduation days to prepare them for clearing various competitive examinations, campus placements, entrance examinations. Hence, this participatory book on Employability Skills will be a guide for students to learn about the concepts and enhance their aptitude skills.

All the Best!
KSHCE and KSDC

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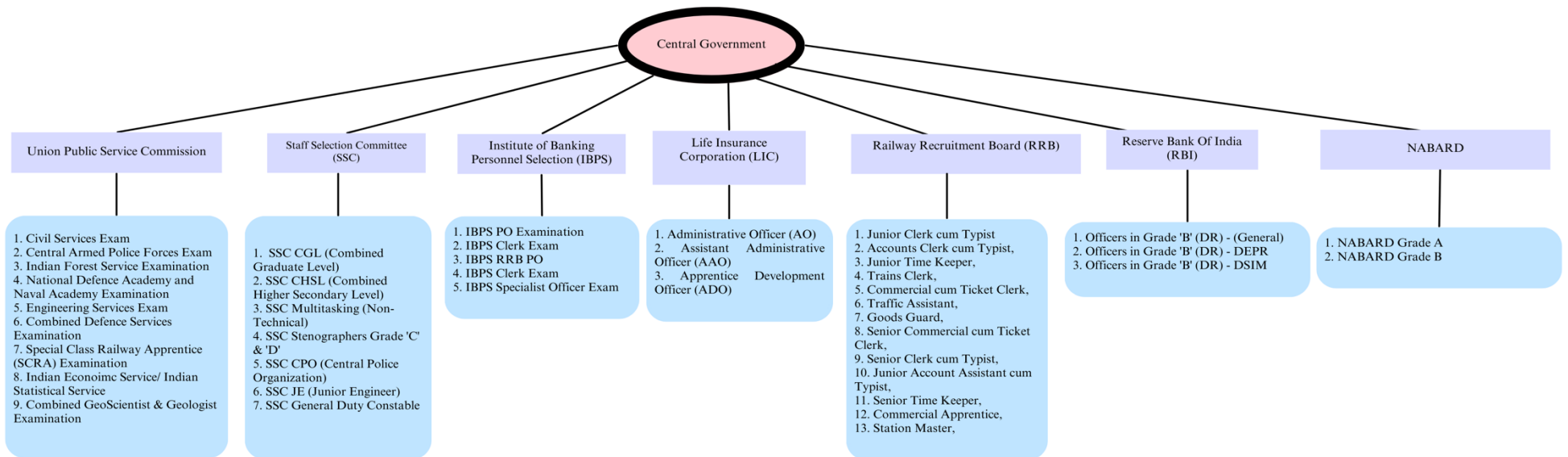
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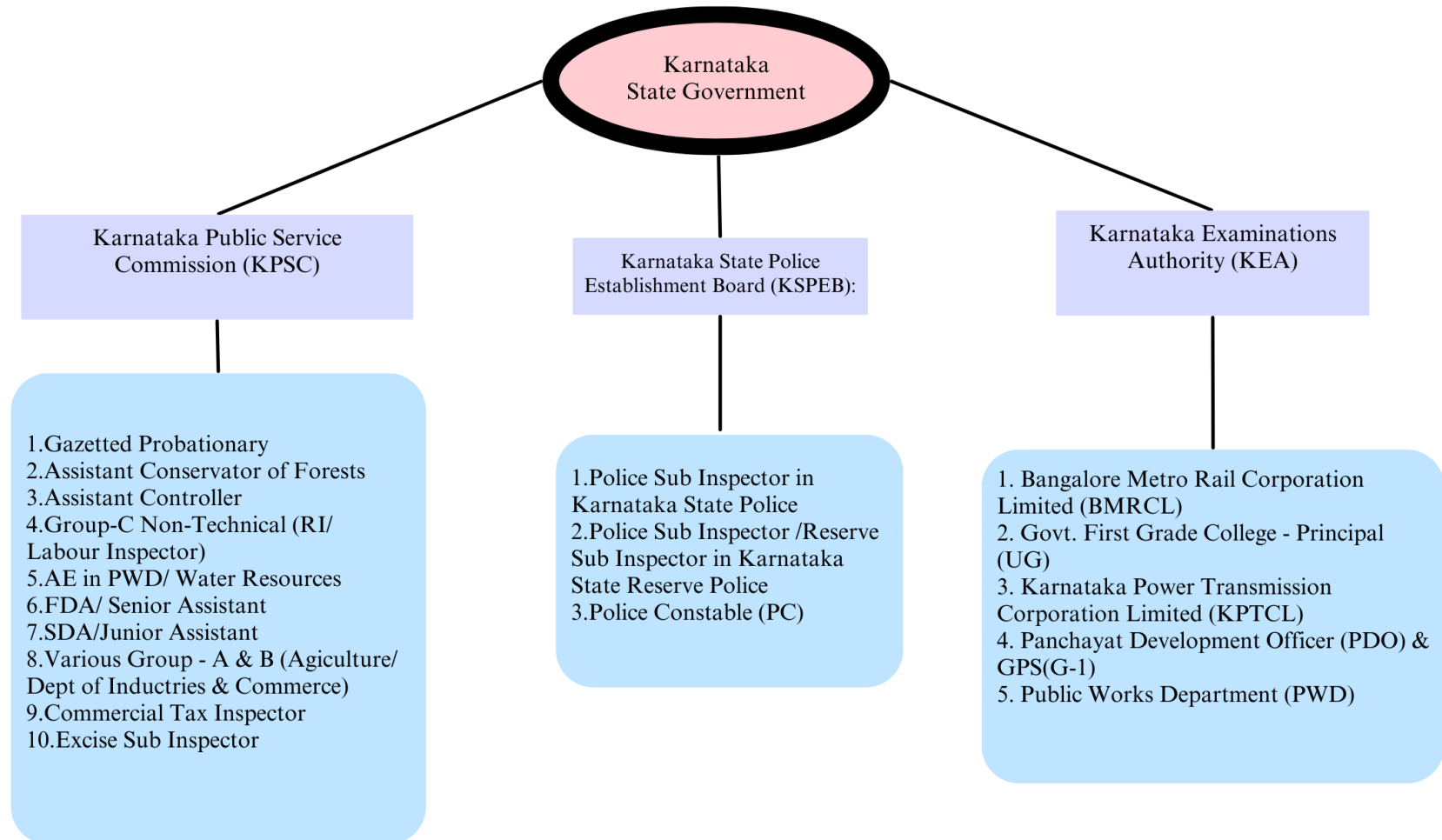
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List of Competitive Examinations

Central Government



Karnataka Government Competitive Examinations



Sl.No	Name of the Competitive Exam	Conducted By	Eligibility Criteria Age in years (Upper Age Cap)					Minimum Educational Qualification	Other requirements	Exam Pattern (number of stages and their weightage in the selection process, etc)			Syllabus
			GM	OBC	SC	ST	PwD						
1	Civil Services Exam	UPSC	32	35	37	37	5/10 Years relaxation	Any Degree	Physical Standards criteria for technical services	Prelims	Mains	Personality Test	Prelims - General Studies& CSAT/ Mains - GeneralStudies, Essay, Compulsory papers & Optional subjects
2	Central Armed Police Forces - ACExam	UPSC	25	28	30	30		Any Degree	Physical Standards criteria	Written exam	Physical efficiency& Medical	Personality Test	Written Exam - General Studies & English
3	Indian Forest Service Examination	UPSC	32	35	37	37	5/10 Years relaxation	Degree - Science, Engineering & Veterinary Graduates	Physical Standards criteria	Prelims	Mains	Personality Test	Prelims - General Studies& CSAT Mains - GeneralStudies, Compulsory papers & Optional subjects
4	National Defence Academy and Naval Academy Examination	UPSC	16.5-19.5	16.5-19.5	16.5-19.5	16.5-19.5		12th Pass	Physical standards	Written	Intelligence & Personality Test	-	General Ability Test & Mathematics
5	Engineering Services Exam	UPSC	30	33	35	35	5 Years relaxation	Bachelors in engineering/Technology	-	Prelims	Mains	Personality Test	General Studies and respective Engineering Subjects
6	Combined Defence Services Examination	UPSC	19-25	19-25	19-25	19-25		Any Degree/ For Airforce - Science & Engineering	Unmarried Males and for females unmarried/widow/divo & Physical	Mains	Intelligence & Personality Test SSB interview	-	General Studies & English
7	Special Class Railway Apprentice(SCRA) Examination	UPSC	17-21	17-24	17-26	17-26	5 Years relaxation	12th Pass	-	Written	SSB interview	-	General Ability Test (English, General Knowledge, and psychological test), Physical Science (Physics and Chemistry) and Mathematics
8	Indian Economic Service/ Indian Statistical Service	UPSC	30	33	35	35	10 Years relaxation	Graduation/ Post Graduation	-	Prelims	Mains	Personality test	General Studies, General English, General Economics & Indian Economics
9	Combined Geoscientist & Geologist Examination	UPSC	32	35	38	38	10 Years relaxation	Graduation/ Post Graduation	-	Written	Personality test	-	General Studies, General English, Geology, Geophysics & Chemistry
10	SSC CGL (Combined Graduate Level)	SSC	27	30	32	32	10 Years relaxation	Any Degree	Physical standards	Tier I and Tier II- Computer test	Tier III - written	Tier IV - skill test/Physical test	General Intelligence and Reasoning, General Awareness, Quantitative Aptitude & English Comprehension/ General Studies (Finance & Economics)
11	SSC CHSL (Combined HigherSecondary Level)	SSC	25	28	30	30	10 Years relaxation	12th Pass	Physical standards	Tier I - computer test	Tier II - written	Tier III - skill test/physical test	General Intelligence and Reasoning, GeneralAwareness, Quantitative Aptitude & EnglishComprehension
12	SSC Multitasking (Non-Technical)	SSC	25	28	30	30	10 Years relaxation	12th Pass	Physical standards	Tier I and Tier II - Computer test	Tier III - written	Tier IV - skill test/physical test	General Intelligence and Reasoning, GeneralAwareness, Quantitative Aptitude & EnglishComprehension
13	SSC Stenographers Grade 'C' & 'D'	SSC	30	33	35	35	10 Years relaxation	12th Pass	-	Tier I - computer test	Tier II - skill test	-	General Intelligence and Reasoning, GeneralAwareness, Quantitative Aptitude & EnglishComprehension
14	SSC CPO (Central Police Organization)	SSC	25	28	30	30	10 Years relaxation	Any degree	Physical standards	Written exam	Physical test	Medical exam	General Intelligence and Reasoning, GeneralAwareness, Quantitative Aptitude & EnglishComprehension

SL.N o	Name of the Competitive Exam	Organization By	Eligibility Criteria Age in years (Upper Age Cap)					Minimum Educational Qualification	Other requirements	Exam Pattern (number of stages and their weightage in the selection process, etc)			Syllabus
			GM	OBC	SC	ST	PwD						
15	SSC JE (Junior Engineer)	SSC	30	33	35	35	10 Years relaxation	Diploma or Engineering Degree in relevant field	-	Written exam	Interview	-	General Intelligence and Reasoning, General Awareness, Quantitative Aptitude & English Comprehension and General Engineering of respective subject
16	IBPS PO Examination	IBPS	30	33	35	35	10 Years relaxation	Any Degree	-	Prelims	Mains	Interview	Reasoning and Computer Aptitude, English Language, Data Analysis and Interpretation, General Economy and Banking, Data Analysis & Interpretation, Letter Writing and Essay
17	IBPS Clerk Exam	IBPS	28	30	33	33	10 Years relaxation	Any Degree	-	Prelims	Mains	-	Reasoning and Computer Aptitude, English Language, Data Analysis and Interpretation, General Economy and Banking, Data Analysis & Interpretation, Letter Writing and Essay
18	IBPS RRB PO	IBPS	30	33	35	35	10 Years relaxation	Any Degree	-	Prelims	Mains	Interview	Reasoning and Computer Aptitude, English Language, Data Analysis and Interpretation, General Economy and Banking, Data Analysis & Interpretation, Letter Writing and Essay
19	IBPS Clerk Exam	IBPS	28	30	33	33	10 Years relaxation	Any Degree	-	Prelims	Mains	-	Reasoning and Computer Aptitude, English Language, Data Analysis and Interpretation, General Economy and Banking, Data Analysis & Interpretation, Letter Writing and Essay
20	IBPS Specialist Officer Exam	IBPS	30	33	35	35	10 Years relaxation	Specialist relevant degree/PG in Agriculture, Engineering, Law, IT, Marketing, HR & Post Graduation	-	Prelims	Mains	Personality test	Agriculture, Engineering, Law, IT, Marketing, HR
21	SSC General Duty Constable	SSC	23	26	28	28	10 Years relaxation	12th Pass	Physical standards	Prelims	Physical test	Medical exam	General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary Mathematics & English/ Hindi
22	Gazetted Probationars	KPSC	35	38	40	40	10 Years relaxation	Any Degree	Physical standards	Prelims	Mains	Personality test	Prelims - General Studies & CSAT / Mains - General Studies & Essay, Compulsory papers
23	Assistant Conservator of Forests	KPSC	30	33	35	35	10 Years relaxation	Degree - Science, Engineering & Veterinary Graduates	Physical standards	Prelims	Mains	Personality test	Prelims - General Studies & CSAT / Mains - Optional subjects & Aptitude
24	Assistant Controller	KPSC	35	38	40	40	10 Years relaxation	PG in Commerce or Finance		Prelims	Mains	Personality test	Commerce & Finance
25	Group-C Non-Technical (Revenue Inspector/ Labour Inspector)	KPSC	35	38	40	40	5 Years relaxation	Any Degree & Commerce for specific departments		Prelims	Document verification		General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary Mathematics, Computer awareness & English/Kannada
26	AE in PWD/ Water Resources	KPSC	35	38	40	40	5 Years relaxation	Engineering in respective subject		Prelims	Document verification		General Studies, Engineering Subject syllabus

SL.N o	Name of the Competitive Exam	Organization By	Eligibility Criteria Age in years (Upper Age Cap)					Minimum Educational Qualification	Other requirements	Exam Pattern (number of stages and their weightage in the selection process, etc)			Syllabus
			GM	OBC	SC	ST	PwD						
27	FDA/ Senior Assistant	KPSC	35	38	40	40	5 Years relaxation	Any Degree		Prelims	Document verification		General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary Mathematics, Computer awareness & English/Kannada
28	SDA/Junior Assistant	KPSC	35	38	40	40	5 Years relaxation	12th Pass		Prelims	Document verification		General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary Mathematics, Computer awareness & English/Kannada
29	PSI/RSI in KSRP	KPESB (Police Establishment Board)	28	30	33	33	10 Years relaxation	Any Degree	Physical standards	Physical Test	Prelims	Medical exam	General Intelligence and Reasoning, General Awareness, Quantitative Aptitude & English Comprehension
30	PC (Police Constable)	KPESB	23	25	27	27	10 Years relaxation	12th Pass	Physical standards	Written exam	Physical test	Medical exam	General Intelligence and Reasoning, General Awareness, Quantitative Aptitude & English Comprehension
31	Various Group - A & B (Agriculture/Dept of Industries & Commerce)	KPSC	35	38	40	40	5 Years relaxation	Degree in respective specialization	-	Prelims	Mains	Personalit y test	General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary Mathematics, Computer awareness, English/Kannada & respective specialization subject (Agriculture & Engineering)
32	Commercial Tax Inspector	KPSC	35	38	40	40	5 Years relaxation	Degree (Maths should be one subject of study)	-	Written exam	Document verification		General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary Mathematics, Computer awareness & English/Kannada
33	LIC AO/AAO/ADO/HFL	LIC	30	33	35	35	10 Years relaxation	Any Degree	-	Prelims	Mains	Personalit y test	General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary Mathematics, Computer awareness, English
34	Administrative officer (Various Insurance Companies)	NIC/NIACL/OI CL/UIA/IRDA	30	33	35	35	10 Years relaxation	Any Degree		Prelims	Mains	Personalit y test	General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary
35	Excise Sub Inspector	KPSC	28	30	33	33	10 Years relaxation	Any Degree	Physical standards	Prelims	Physical test		General Intelligence & Reasoning, General Knowledge & General Awareness, Elementary Mathematics, Computer awareness &
36	Junior Clerk cum Typist	RRB	30	33	35	35	40	Any Degree	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)
37	Accounts Clerk cum Typist	RRB	30	33	35	35	40	12th Pass	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)
38	Junior Timekeeper	RRB	30	33	35	35	40	Any Degree	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)
39	Trains Clerk	RRB	30	33	35	35	40	Any Degree	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)

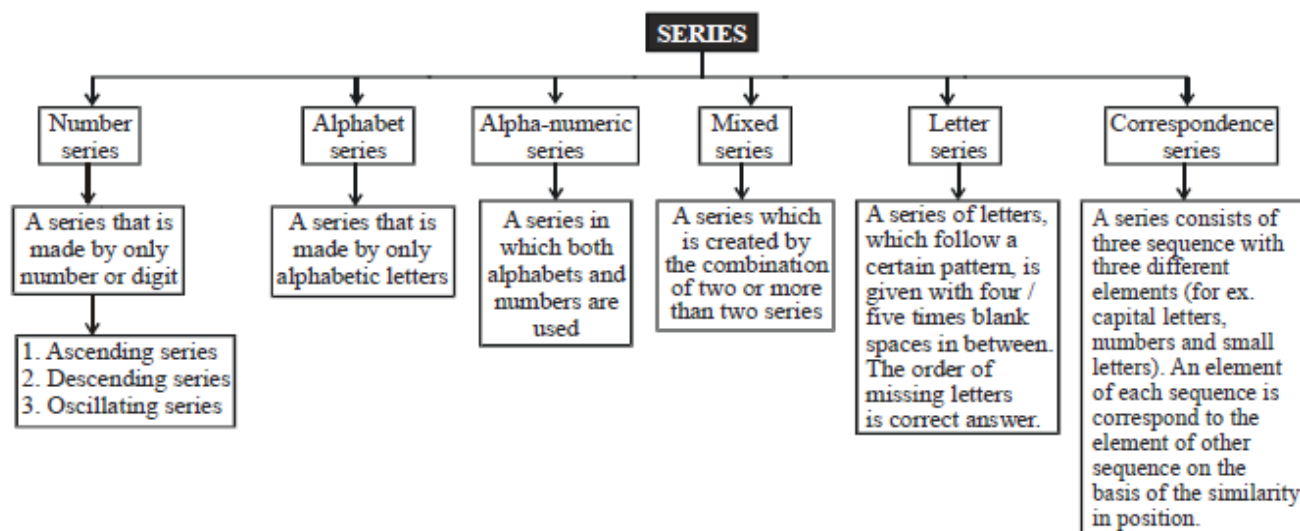
SL.N o	Name of the Competitive Exam	Organization By	Eligibility Criteria Age in years (Upper Age Cap)					Minimum Educational Qualification	Other requirements	Exam Pattern (number of stages and their weightage in the selection process, etc)			Syllabus
			GM	OBC	SC	ST	PwD						
40	Commercial cum Ticket Clerk	RRB	30	33	35	35	40	12th Pass	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)
41	Traffic Assistant	RRB	33	36	35	35	40	Any Degree	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)
42	Goods Guard	RRB	33	36	35	35	40	Any Degree	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)
43	Senior Commercial cum Ticket Clerk	RRB	33	36	35	35	40	Any Degree	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)
44	Senior Clerk cum Typist	RRB	33	36	35	35	40	Any Degree	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning General studies Typing test (30 words per min)
45	Junior Account Assistant cum Typist	RRB	33	36	35	35	40	Any Degree	-	Tier I CBT	Tier II CBT	Typing Test	General Mathematics Reasoning
46	NABARD Grade A	NABARD	30	33	35	35	40	Any Degree	-	Prelims	Mains	Interview	General Mathematics Reasoning General studies
47	NABARD Grade B	NABARD	32	35	37	37	42	Any Degree	-	Prelims	Mains	Interview	General Mathematics Reasoning General studies
48	RBI Grade B (DR) - (General)	RBI	30	33	35	35	40	Any Degree	-	Prelims	Mains	Interview	General Mathematics Reasoning General studies
49	RBI Grade B (DR) - DEPR	RBI	30	33	35	35	40	Post-Graduation	-	Prelims	Mains	Interview	General Mathematics Reasoning General studies
50	RBI Grade B (DR) - DSIM	RBI	30	33	35	35	40	Post-Graduation	-	Prelims	Mains	Interview	General Mathematics Reasoning General studies
	Bangalore Metro Rail Corporation Limited (BMRL) Station Controller, Train Operator	KEA	35	38	40	40	-	-	-	Prelims	Mains	-	General Science Electronics and Communication Mechanical
53	Govt. First Grade College - Principal (UG)	KEA	40	45	45	45	50	Phd.	15 Years Experience	Prelims	-	-	General Mathematics Reasoning General studies
SS	Karnataka Power Transmission Corporation Limited (KPTCL)		35	38	40	40	45	BE, Degree, Civil	-	Prelims	Interview	-	General Awareness Reasoning Technical paper VTU syllabus
63	Panchayat Development Officer (PDO) and GPS(G-1)	KEA	35	38	40	40	50	Any Degree	-	Prelims	Interview	-	General Reasoning General English General Kannada

64	Public Works Department (PWD) Assistant Engineer	KEA	35	38	40	45	45	BE (Civil/ Electrical)	-	Prelims	Interview	-	General Awareness Reasoning, Quantitative Aptitude Technical Paper VTU syllabus
64	Public Works Department (PWD) Junior Engineer	KEA	35	38	40	45	45	Diploma (Civil/ Electrical)	-	Prelims	Interview	-	General Awareness Reasoning, Quantitative Aptitude Technical Paper Diploma syllabus

Chapter 01: Series

A series is a sequence of numbers/alphabetical letters or both which follow a particular rule. Each element of series is called 'term'. We have to analyse the pattern and find the missing term or next term to continue the pattern.

Types of Series are explained in the following chart :



Number Series, Relationship Between the Terms of Number Series

For example.

- Consecutive even numbers
- Consecutive odd numbers
- Consecutive prime numbers
- Square of numbers
- Cubes of numbers
- Square root of numbers
- Omission of certain number of letter in any consecutive order
- Addition /subtraction/ multiplication/ division by some number (For Example : A.P & G.P) or any other relation.

TYPE OF NUMBER SERIES

(I) Complete the series

Example 1 4, 6, 9, 13, ...

- (a) 17 (b) 18
(c) 19 (d) 20

Solutions. (b) 4 6 9 13 **18** Correct answer
 +2 +3 +4 +5

Example 2 64, 32, 16, 8, ?

- (a) 0 (b) 1
(c) 2 (d) 4

Solutions. (d) Each number is half of its previous number.

Example 3 4, 9, 16, 25, ?

- (a) 32 (b) 42
(c) 55 (d) 36

Solutions. (d) Each number is a whole square.

Example 4 2, 6, 12, 20, 30, 42, 56, ?

- (a) 60 (b) 64
(c) 70 (d) 72

Solutions. (d) $1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5, 5 \times 6, 6 \times 7, 7 \times 8, 8 \times 9 = 72$

(II) To find the missing number of series :

Example 5 79, 87, ?, 89, 83

- (a) 80, (b) 81
(c) 82 (d) 88

Solutions. (b) 79 87 81 89 83
+8 -6 +8 -6

Example 6 37, 41, ?, 47, 53

- (a) 42 (b) 43
(c) 46 (d) 44

Solutions. (b) Consecutive prime numbers.

Example 7 21, 34, ?, 89, 144

- (a) 43 (b) 55
(c) 64 (d) 71

Solutions. (b) Each number is the sum of the two preceding numbers.

$$21 + 34 = 55$$

$$34 + 55 = 89$$

$$55 + 89 = 144$$

(III) To find the wrong term in the series :

Example 8 3, 8, 15, 24, 34, 48, 63.

- (a) 15 (b) 15
(c) 34 (d) 63

Solutions. (c) $2^2 - 1$, $3^2 - 1$, $4^2 - 1$, $5^2 - 1$, $6^2 - 1$

ALPHABET SERIES

A series that is made of only alphabetic letters is called alphabetic Series

Examples of Alphabetic Series

Example 9 G, H, J, M, ?

- (a) R (b) S
(c) Q (d) P

Solutions. (c) G H J M Q
+1 +2 +3 +4

Example 10 BF, CH, ?, HO, LT

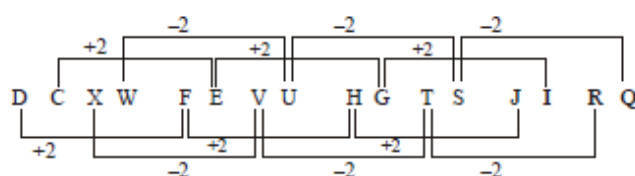
- (a) FG (b) HK
(c) CE (d) FJ

Solutions. (b) BF CH EK HO LT
+1 +2 +3 +4

Example 11 DCXW, FEVU, HGTS, ?

- (a) LKPO (b) ABYZ
(c) JIRQ (d) LMRS

Solutions. (c) JIRQ



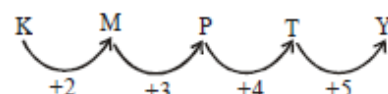
ALPHA-NUMERIC SERIES

A series in which both alphabet and number are used called alpha-numeric series.

Example 12 K 1, M 3, P 5, T 7, ?

- (a) Y 9 (b) Y 11
(c) V 9 (d) V 11

Solutions. (b) Alphabets follow the sequence



And numbers are increasing by 2

Example 13 Find the missing term.

2 Z 5, 7 Y 7, 14 X 9, 23 W 11, 34 V 13, ?

- (a) 47 T 15 (b) 47 U 17
(c) 47 U 15 (d) 48 U 15

Solutions. (c) First number is the sum of the numbers of the preceding term.

Middle letter is moving one step backward.

Third number in a term is a series of odd numbers.

$\therefore$ 6th term = 47 U 15.

MIXED SERIES

A series which is created by the combination of two or more than two series called mixed series.

Example 14 Complete the series

Z, L, X, J, V, H, T, F, ?, ?

- (a) D, R (b) R, D
(c) D, D (d) R, R

Solutions. (b) The given sequence consists of two series

(i) Z, X, V, T, __

(ii) L, J, H, F, __. Both consisting of alternate letters in the reverse order.

$\therefore$ Next term of (i) series = R, and

Next term of (ii) series = D

Example 15 7, 5, 26, 17, 63, 37, 124, 65, ?, ?

- (a) 101, 215 (b) 101, 101
(c) 215, 101 (d) 215, 215

Solutions. (c) The given series consists of two series

(i) 7, 26, 63, 124, ...

(ii) 5, 17, 37, 65, ...

In the first series,

$$7 = 2^3 - 1, 26 = 3^3 - 1, 63 = 4^3 - 1,$$

$$124 = 5^3 - 1, \therefore 6^3 - 1 = 215$$

and in the second series.

$$5 = 2^2 + 1, 17 = 4^2 + 1,$$

$$37 = 6^2 + 1, 65 = 8^2 + 1,$$

$$\therefore 10^2 + 1 = 101$$

LETTER SERIES

A series of letters, which follow a certain pattern, is given with four/ five times blank spaces in between. The order of missing letters is correct answer.

Example 16 b a a b - a b a - b b a - -

(a) bbaa (b) aaaa

(c) abab (d) baba

Solutions. (d) b a a b b a / b a a b b a / b a.

Example 17 - - a a b - a - a - b a

(a) bbaab (b) ababa

(c) bbabb (d) aaaba

Solutions. (b) a b a / a b a / a b a / a ba.

CORRESPONDENCE SERIES

A series consists of three sequence with three different elements (for example capital letters, numbers and small letters). An element of each sequence is correspond to the element of other sequence on the basis of the similarity in position.

Example 18 Find the missing term (?) in the following sequence.

A	—	B	A	C	—	D	—	B	C	D	C
—	3	—	2	—	1	—	4	?	?	?	?
d	c	—	—	b	a	c	b	—	—	—	—

(a) 1, 3, 4, 3

(b) 1, 4, 3, 4

(c) 2, 3, 4, 3

(d) 3, 4, 1, 4

Solutions. (b) Clearly, 2 corresponds to A.

Now, b corresponds to C and 4 corresponds to b. So, 4 corresponds to C.

c corresponds to D and 3 corresponds to c. So, 3 corresponds to D.

So, the remaining number i.e., 1 corresponds to B.

Thus, BCDC corresponds to 1, 4, 3, 4.

Example 19 Find the missing term (?) in the following sequence.

C	B	—	—	D	—	B	A	B	C	C	B
—	—	1	2	4	3	—	—	?	?	?	?
a	—	a	b	—	c	—	b	—	—	—	—

(a) 3, 4, 4, 3

(b) 3, 2, 2, 3

(c) 3, 1, 1, 3

(d) 1, 4, 4, 1

Solutions. (c) Comparing the positions of the capital letters, numbers and small letters, we find :

a corresponds to C and 1 corresponds to a. So, a and 1 correspond to C.

b corresponds to A and 2 corresponds to b. So, b and 2 correspond to A.

Also, 4 corresponds to D.

So, the remaining number i.e. 3 corresponds to B. So, BCCB corresponds to 3, 1, 1, 3.

Chapter 02: Number System

NUMBER SYSTEM

The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *digits*, which can represent any number.

Number System is a mathematical structure which includes natural numbers, whole numbers, real numbers, rational numbers, irrational numbers, etc.

NATURAL NUMBERS : These are the numbers (1, 2, 3, 4, ...) that are used for counting. It is denoted by N . There are infinite natural numbers and the smallest natural number is 1.

EVEN NUMBERS : Natural numbers which are divisible by 2 are even numbers. It is denoted by E .

$E = 2, 4, 6, 8, \dots$

Smallest even number is 2. There is no largest even number.

ODD NUMBERS : Natural numbers which are not divisible by 2 are odd numbers. It is denoted by O .

$O = 1, 3, 5, 7, \dots$

Smallest odd number is 1. There is no largest odd number.

PRIME NUMBERS : Natural numbers which have exactly two factors, i.e., 1 and the number itself are called prime numbers.

e.g. 2, 3, 5, 7, ...

The lowest prime number is 2. 2 is also the only even prime number.

RELATIVE PRIME NUMBER : Two numbers are said to be relatively prime if they do not have any common factor other than 1. e.g. (3, 5), (5, 7) etc. (11, 15) etc.

TWIN PRIMES : Two prime numbers which differ by 2 are called twin prime number. e.g. 4 (3, 5), (5, 7), (11, 13)

COMPOSITE NUMBERS : It is a natural number that has at least one divisor different from unity and itself.

Every composite number can be factorised into its prime factors.

For Example : $24 = 2 \times 2 \times 2 \times 3$. Hence, 24 is a composite number.

The smallest composite number is 4.

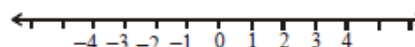
WHOLE NUMBERS : The natural numbers along with zero (0), form the system of whole numbers. It is denoted by W .

There is no largest whole number and the smallest whole number is 0.

INTEGERS : The number system consisting of natural numbers, their negative and zero is called integers. It is denoted by Z or I .

The smallest and the largest integers cannot be determined.

THE NUMBER LINE : The number line is a straight line between negative infinity on the left to positive infinity on the right.



REAL NUMBERS : All numbers that can be represented on the number line are called real numbers. It is denoted by R .

R^+ : Positive real numbers and

R^- : Negative real numbers.

Real numbers = Rational numbers + Irrational numbers.

RATIONAL NUMBERS : Any number that can be put in the form

of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational

number. It is denoted by Q .

Every integer is a rational number. Zero (0) is also a rational number.

The smallest and largest rational numbers cannot be determined.

Every fraction (and decimal fraction) is a rational number.

$$Q = \frac{p \text{ (Numerator)}}{q \text{ (Denominator)}}$$

BASIC RULES ON NATURAL NUMBERS

- One digit numbers are from 1 to 9. There are 9 one digit numbers. i.e. 9×10^0 .
- Two digit numbers are from 10 to 99. There are 90 two digit number. i.e. 9×10^1 .
- Three digit numbers are from 100 to 999. There are 900 three digit numbers. i.e. 9×10^2 .

In general, the number of n digit numbers are $9 \times 10^{(n-1)}$

- Sum of first n natural number $= 1 + 2 + 3 + \dots + n = \frac{(n)(n+1)}{2}$
- Sum of squares of first n natural numbers

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{(n)(n+1)(2n+1)}{6}$$

- Sum of cubes of first n natural number $=$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{(n)(n+1)}{2} \right)^2$$

If x and y are two rational numbers, then $\frac{x+y}{2}$ is also a rational

number and its value lies between the given two rational numbers x and y .

An infinite number of rational numbers can be determined between any two rational numbers.

Example 1 Find three rational numbers between 3 and 5.

Solution :

$$1\text{st rational number} = \frac{3+5}{2} = \frac{8}{2} = 4$$

2nd rational number (i.e., between 3 and 4)

$$= \frac{3+4}{2} = \frac{7}{2}$$

3rd rational number (i.e., between 4 and 5)

$$= \frac{4+5}{2} = \frac{9}{2}$$

IRRATIONAL NUMBERS : The numbers which are not rational or which cannot be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called irrational number. It is denoted by Q' or Q^c .

$\sqrt{2}, \sqrt{3}, \sqrt{5}, 2+\sqrt{3}, 3-\sqrt{5}, 3\sqrt{3}$ are irrational numbers.

Every positive irrational number has a negative irrational number corresponding to it.

$$\sqrt{2} + \sqrt{3} \neq \sqrt{5}$$

$$\sqrt{5} - \sqrt{3} \neq \sqrt{2}$$

$$\sqrt{3} \times \sqrt{2} = \sqrt{3 \times 2} = \sqrt{6}$$

$$\sqrt{6} \div \sqrt{2} = \sqrt{\frac{6}{2}} = \sqrt{3}$$

Some times, product of two irrational numbers is a rational number.

$$\text{For example : } \sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = 2$$

$$(2 + \sqrt{3}) \times (2 - \sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$$

Both rational and irrational numbers can be represented on number line. Thus real numbers is the set of the union of rational and irrational numbers.

$$R = Q \cup Q'$$

Every real number is either rational or irrational.

FRACTION : A fraction is a quantity which expresses a part of the whole.

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

Example 2 Write a fraction whose numerator is $2^2 + 1$ and denominator is $3^2 - 1$.

Solution :

$$\text{Numerator} = 2^2 + 1 = 4 + 1 = 5$$

$$\text{Denominator} = 3^2 - 1 = 9 - 1 = 8$$

$$\therefore \text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}} = \frac{5}{8}$$

TYPES OF FRACTIONS :

1. **Proper Fraction :** If numerator is less than its denominator, then it is a proper fraction.

$$\text{For example : } \frac{2}{5}, \frac{6}{18}$$

2. **Improper Fraction :** If numerator is greater than or equal to its denominator, then it is an improper fraction.

$$\text{For example : } \frac{5}{2}, \frac{18}{7}, \frac{13}{13}$$

If in a fraction, its numerator and denominator are of equal value then fraction is equal to unity i.e., 1.

3. **Mixed Fraction :** It consists of an integer and a proper fraction.

$$\text{For example : } 1\frac{1}{2}, 3\frac{2}{3}, 7\frac{5}{9}$$

Mixed fraction can always be changed into improper fraction and vice versa.

$$\text{For example : } 7\frac{5}{9} = \frac{7 \times 9 + 5}{9} = \frac{63 + 5}{9} = \frac{68}{9} \text{ and}$$

$$\frac{19}{2} = \frac{9 \times 2 + 1}{2} = 9 + \frac{1}{2} = 9\frac{1}{2}$$

4. **Equivalent fractions / Equal fractions :** Fractions with same value are called equivalent fraction.

$$\text{For example : } \frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$$

Value of fraction is not changed by multiplying or dividing the numerator or denominator by the same number.

For example :

$$(i) \quad \frac{2}{5} = \frac{2 \times 5}{5 \times 5} = \frac{10}{25} \quad \text{So, } \frac{2}{5} = \frac{10}{25}$$

$$(ii) \quad \frac{36}{16} = \frac{36 \div 4}{16 \div 4} = \frac{9}{4} \quad \text{So, } \frac{36}{16} = \frac{9}{4}$$

5. **Like Fractions :** Fractions with same denominators are called like fraction.

$$\text{For example : } \frac{2}{7}, \frac{3}{7}, \frac{9}{7}, \frac{11}{7}$$

6. **Unlike Fractions** : Fractions with different denominators are called unlike fraction.

For example : $\frac{2}{5}, \frac{4}{7}, \frac{9}{8}, \frac{9}{2}$

Unlike fractions can be converted into like fractions.

For example : $\frac{3}{5}$ and $\frac{4}{7}$

$$\frac{3}{5} \times \frac{7}{7} = \frac{21}{35} \text{ and } \frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$$

7. **Simple Fraction** : If in a fractions, numerator and denominator are integers then it is called simple fraction.

For example : $\frac{3}{7}$ and $\frac{2}{5}$.

8. **Complex Fraction** : If in a fractions, Numerator or denominator or both are fractional numbers, then it is called complex fraction.

For example : $\frac{2}{\frac{5}{7}}, \frac{2\frac{1}{3}}{5\frac{2}{3}}, \frac{2+\frac{1}{7}}{2+\frac{3}{2}}$

9. **Decimal Fraction** : Denominator with the powers of 10, then it is a decimal fraction.

For example : $\frac{2}{10} = (0.2), \frac{9}{100} = (0.09)$

10. **Vulgar Fraction** : If denominator of any fraction is not the power of 10, then it is called vulgar fraction.

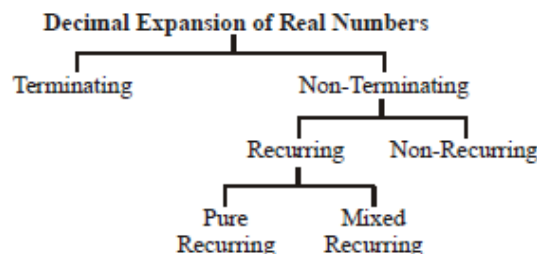
For example : $\frac{3}{7}, \frac{9}{2}, \frac{5}{193}$.

Rounding off (Approximation) of Decimals : There are some decimals in which numbers are found upto large number of decimal places.

For example : 3.4578, 21.358940789.

But many times we require decimal numbers upto a certain number of decimal places. Therefore,

If the digit of the decimal place is five or more than five, then the digit in the preceding decimal place is increased by one and if the digit in the last place is less than five, then the digit in the preceding place remains unchanged.



Terminating (or finite decimal fractions) :

For example : $\frac{7}{8} = 0.875, \frac{21}{5} = 4.2$.

Non-terminating decimal fractions : There are two types of Non-terminating decimal fractions.

- (i) **Non-terminating periodic fractions or non-terminating recurring (repeating) decimal fractions :**

Form : $x.a_1a_2a_3 \dots a_1a_2a_3 \dots a_1a_2a_3$

For example : $\frac{10}{3} = 3.333 \dots = 3\bar{3}$

$$\frac{1}{7} = 0.142857142857\dots = 0.\overline{142857}$$

- (ii) **Non-terminating non-periodic fraction or non-terminating non-recurring fractions :**

Form : $x.a_1a_2a_3 \dots b_1b_2b_3 \dots c_1c_2c_3 \dots$

For example : 15.2731259629

The decimal expansion of a rational number is either terminating or non-terminating recurring. Moreover, a number whose decimal expansion is terminating or non-terminating recurring is rational.

The decimal expansion of an irrational number is non-terminating non recurring. Moreover, a number whose decimal expansion is non-terminating non recurring is irrational.

For example : $\sqrt{2} = 1.41421356237309504880\dots$

$$\pi = 3.1415926535897932384626433\dots$$

We often take $\frac{22}{7}$ as an approximate value of π , but $\pi \neq \frac{22}{7}$.

Example 3 Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

Solution :

We find by dividing, $\frac{1}{7} = 0.\overline{142857}$ and $\frac{2}{7} = 0.\overline{285714}$.

To find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$, we find a number which is non-terminating non-recurring lying between them.

So, 0.1501500150000... is an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

COMPLEX NUMBERS : A number of the form $a + bi$, where a and b are real number and $i = \sqrt{-1}$ (imaginary number) is called a complex number. It is denoted by C .

For Example : $5i$ ($a = 0$ and $b = 5$), $\sqrt{5} + 3i$ ($a = \sqrt{5}$ and $b = 3$)

DIVISIBILITY RULES

Divisibility by 2 : A number is divisible by 2 if its unit's digit is even or 0.

Divisibility by 3 : A number is divisible by 3 if the sum of its digits are divisible by 3.

Divisibility by 4 : A number is divisible by 4 if the last 2 digits are divisible by 4, or if the last two digits are zeros.

Divisibility by 5 : A number is divisible by 5 if its unit digit is 5 or 0.

Divisibility by 6 : A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisibility by 7 : A number is divisible by 7 if unit's place digit is multiplied by 2 and subtracted from the remaining digits and the number obtained is divisible by 7.

Divisibility by 8 : A number is divisible by 8 if the last 3 digits of the number are divisible by 8, or if the last three digits of a number are zeros.

Divisibility by 9 : A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility by 10 : A number is divisible by 10 if its unit digit is 0.

Divisibility by 11 : A number is divisible by 11 if the sum of digits at odd and even places are equal or differ by a number divisible by 11.

Divisibility by 12 : A number is divisible by 12 if the number is divisible by both 4 and 3.

Divisibility by 13 : A number is divisible by 13 if its unit digit is multiplied by 4 and added to the remaining digits and the number obtained is divisible by 13.

Divisibility by 14 : A number is divisible by 14 if the number is divisible by both 2 and 7.

Divisibility by 15 : A number is divisible by 15 if the number is divisible by both 3 and 5.

Divisibility by 16 : A number is divisible by 16 if its last 4 digits is divisible by 16 or if the last four digits are zeros.

Divisibility by 17 : A number is divisible by 17 if its unit digit is multiplied by 5 and subtracted from the remaining digits and the number obtained is divisible by 17.

Divisibility by 18 : A number is divisible by 18 if the number is divisible by both 2 and 9.

Divisibility by 19 : A number is divisible by 19 if its unit digit is multiplied by 2 and added to the remaining digits and the number obtained is divisible by 19.

Example 4 Without actual division, find which of the following numbers are divisible by 2, 3, 4, 5, 7, 9, 10, 11 :

- (i) 36324 (ii) 2211 (iii) 87120

Solution : (i) 36324

It is divisible by 2 because 4 (unit digit) is divisible by 2.

It is divisible by 3 because $3 + 6 + 3 + 2 + 4 = 18$ is divisible by 3.

It is divisible by 4 because 24 is divisible by 4.

It is not divisible by 5.

It is not divisible by 7.

It is divisible by 9 because $3 + 6 + 3 + 2 + 4 = 18$ is divisible by 9.

It is not divisible by 10.

It is not divisible by 11.

(ii) 2211

It is not divisible by 2.

It is divisible by 3 because $2 + 2 + 1 + 1 = 6$ is divisible by 3.

It is not divisible by 4, 5, 7, 8, 10.

It is divisible by 11 because $2211 \rightarrow (2 + 1) - (2 + 1) = 3 - 3 = 0$.

(iii) 87120

It is divisible by 2 because its unit's place digit is 0.

It is divisible by 3 because $8 + 7 + 1 + 2 + 0 = 18$ is divisible by 3.

It is divisible by 4 because 20 is divisible by 4.

It is divisible by 5 because its unit's place digit is 0.

It is not divisible by 7.

It is divisible by 9 because $8 + 7 + 1 + 2 + 0 = 18$ is divisible by 9.

It is divisible by 10 because its unit's place digit is 0.

It is divisible by 11 because $87120 \rightarrow (8 + 1 + 0) - (7 + 2) = 9 - 9 = 0$.

Example 5 Is 473312 divisible by 7 ?

Solution :

$$47331 - 2 \times 2 = 47327$$

$$4732 - 2 \times 7 = 4718$$

$$471 - 2 \times 8 = 455$$

$$45 - 2 \times 5 = 35$$

35 is divisible by 7, therefore, 473312 is divisible by 7.

Example 6 What is the value of M and N respectively if $M39048458N$ is divisible by 8 and 11, where M and N are single digit integers ?

Solution : A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

i.e., $58N$ is divisible by 8.

Clearly, $N = 4$

Again, a number is divisible by 11 if the difference between the sum of digits at even places and sum of digits at the odd places is either 0 or is divisible by 11.

$$\text{i.e., } (M + 9 + 4 + 4 + 8) - (3 + 0 + 8 + 5 + N)$$

$$= M + 25 - (16 + N)$$

$$= M - N + 9 \text{ must be zero or it must be divisible by 11}$$

$$\text{i.e., } M - N = 2$$

$$\Rightarrow M = 2 + 4 = 6$$

$$\text{Hence, } M = 6, N = 4$$

To find the last digit or digit at the unit's place of a^n .

1. If the last digit or digit at the unit's place of a is 1, 5 or 6, whatever be the value of n , it will have the same digit at unit's place, i.e.,

$$(\dots 1)^n = (\dots 1)$$

$$(\dots 5)^n = (\dots 5)$$

$$(\dots 6)^n = (\dots 6)$$

2. If the last digit or digit at the units place of a is 2, 3, 5, 7 or 8, then the last digit of a^n depends upon the value of n and follows a repeating pattern in terms of 4 as given below :

n	last digit of $(\dots 2)^n$	last digit of $(\dots 3)^n$	last digit of $(\dots 7)^n$	last digit of $(\dots 8)^n$
$4x+1$	2	3	7	8
$4x+2$	4	9	9	4
$4x+3$	8	7	3	2
$4x$	6	1	1	6

3. If the last digit or digit at the unit's place of a is either 4 or 9, then the last digit of a^n depends upon the value of n and follows repeating pattern in terms of 2 as given below.

n	last digit of $(\dots 4)^n$	last digit of $(\dots 9)^n$
$2x$	6	1
$2x+1$	4	9

Example 7 What is the unit digit of $(437)^{65} \times (25)^{71}$?

Solution :

unit digit in the given product
 $=$ unit digit in the product $(7^{65} \times 5^{71})$
 Now, $7^{65} = 7^{4(16)+1}$ gives unit digit 7 and 5^{71} gives unit digits
 $\therefore$ Required unit digits in $(7 \times 5) = 35$ is 5.

Example 8 What is the unit digit of $\{(264)^{102} \times (264)^{103}\}$?

Solution :

$(264)^{102} + (264)^{103} = (264)^{102} \{1+264\} = (264)^{102} \times 265$
 Required unit digit = unit digit in $[(4)^{102} \times 5]$
 $=$ unit digit in $[(4^4)^{25} + 4^2 \times 5]$
 $=$ unit digit in $(6 \times 6 \times 5) = 0$

IMPORTANT FACTS

- If p and q are co-primes and both are factors of a number k , then their product $p \times q$ will also be factor of k .
 Factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24, prime factors of 24 are 2 and 3, which are co-primes also. Product of $2 \times 3 = 6$ which is also factor of 24.
- If p divides q and r , then p also divides their sum and difference. e.g. 4 divides 12 and 20. Sum of 12 and 20 is 32 and difference of 12 and 20 is 8. Both are divisible by 4.
- If a number is divisible by another number, then it must be divisible by each of the factors of that number. 48 is divisible by 12. factors of 12 are 1, 2, 3, 4, 6, 12. 48 is divisible by 2, 3, 4 and 6 also.

DIVISION ALGORITHM:

Dividend = (Divisor $\times$ Quotient) + Remainder

where, Dividend = The number which is being divided

Divisor = The number which performs the division process

Quotient = Greatest possible integer as a result of division

Remainder = Rest part of dividend which cannot be further divided by the divisor.

Complete remainder : A complete remainder is the remainder obtained by a number by the method of successive division.

Complete remainder = [I divisor $\times$ II remainder] + I remainder

$$C.R. = d_1 r_2 + r_1$$

$$C.R. = d_1 d_2 r_3 + d_1 r_2 + r_1$$

Two different numbers x and y when divided by a certain divisor D leave remainder r_1 and r_2 respectively. When the sum of them is divided by the same divisor, the remainder is r_3 . Then,

$$\text{divisor } D = r_1 + r_2 - r_3$$

Example 9 A certain number when Successively divided by 3 and 5 leaves remainder 1 and 2. What is the remainder if the same number be divided by 15?

Solution :

Let x be the dividend.

$$x = 3y + 1 \quad \dots(i)$$

now, y become dividend for 5.

$$y = 5z + 2 \quad \dots(ii)$$

putting y in (i)

$$x = 3(5z + 2) + 1$$

$$= 15z + 6 + 1 = 15z + 7$$

when x is divided by 15 gives remainder 7. **Alternate Method :**

$$d_1 = 3, d_2 = 5, r = 1 \text{ and } r_2 = 2$$

$$\text{complete remainder} = d_1 r_2 + r_1 = 3 \times 2 + 1 = 7$$

Example 10 A certain number when divided by 899 leaves the remainder 63. Find the remainder when the same number is divided by 29.

Solution :

$$\begin{aligned} \text{(a) Number} &= 899Q + 63, \text{ where } Q \text{ is quotient} \\ &= 31 \times 29Q + (58 + 5) = 29(31Q + 2) + 5 \\ \therefore \text{Remainder} &= 5 \end{aligned}$$

Method to find the number of different divisors (or factors) (including 1 and itself) of any composite number N :

STEPI : Express N as a product of prime numbers as

$$N = x^a \times y^b \times z^c \dots\dots\dots$$

STEP II : Number of different divisors (including 1 and itself)

$$= (a+1)(b+1)(c+1) \dots\dots\dots$$

Example 11 Find the number of different divisors of 50, besides unity and the number itself.

Solution : If you solve this problem without knowing the rule, you will take the numbers in succession and check the divisibility. In doing so, you may miss some numbers. It will also take more time.

Different divisors of 50 are : 1, 2, 5, 10, 25, 50

If we exclude 1 and 50, the number of divisors will be 4.

$$\text{By rule : } 50 = 2 \times 5 \times 5 = 2^1 \times 5^2$$

$$\therefore \text{the number of total divisors} = (1+1) \times (2+1) = 2 \times 3 = 6$$

$$\text{or, the number of divisors excluding 1 and 50} = 6 - 2 = 4$$

Chapter 03: LCM and HCF

FACTORS : A number may be made by multiplying two or more other numbers together. The numbers that are multiplied together are called factors of the final number.

Factors of 12 = 1, 2, 3, 4, 6, and 12.

All the numbers have a factor of one.

Common Factor : A common factor of two or more given numbers is a number which divides each given number completely.

Common factor of 12 and 18 are 1, 2, 3, 6.

Co-prime Numbers : Two or more numbers that do not have a common factor are known as co-prime or relatively prime.

For example : 4 and 15 are Co-prime numbers.

Highest Common Factor : The highest common factor (H.C.F.) of two or more numbers is the greatest number which divides each of them exactly. It is also known as greatest common divisor (G.C.D.).

H.C.F. can be calculated by :

- (i) Prime Factorisation Method
- (ii) Division Method
- (i) **H.C.F. by Prime Factorisation Method :**

Example 1 Find the H.C.F. of 40 and 60 by prime factorisation method.

$$\begin{array}{rcl} \text{Solution : } 40 & = & 2 \times 2 \times 2 \times 5 \\ & \uparrow \uparrow \uparrow & \\ & 2 \times 2 \times 3 \times 5 & \text{(Common factor)} \\ 60 & = & 2 \times 2 \times 3 \times 5 \end{array}$$

$$\text{Hence H.C.F.} = 2 \times 2 \times 5 = 20.$$

- (ii) **H.C.F. by Division Method :**

Example 2 Find the H.C.F. of 140 and 200 by division method.

$$\begin{array}{r} \text{Solution : } 140 \overline{)200} 1 \\ \underline{140} \\ 60 \\ 60 \overline{)140} 2 \\ \underline{120} \\ 20 \\ 20 \overline{)60} 3 \\ \underline{60} \\ 0 \end{array}$$

$$\therefore \text{H.C.F. of 140 and 200} = 20.$$

H.C.F. of Three Numbers by Division Method :

Example 3 Find the H.C.F. of 324, 630 and 342 by division method.

$$\begin{array}{r} \text{Solution : } 324 \overline{)630} 1 \\ \underline{324} \\ 306 \\ 306 \overline{)324} 1 \\ \underline{306} \\ 18 \\ 18 \overline{)306} 17 \\ \underline{306} \\ 0 \end{array}$$

$$\begin{array}{r} 18 \overline{)342} 19 \\ \underline{342} \\ 0 \end{array}$$

$$\therefore \text{H.C.F. of 324, 630 and 342 is 18.}$$

HCF of Polynomials : When two or more polynomials are factorised, the product of common factor is known as HCF of these polynomials.

For example : Lets find the HCF of $16x^3(x-1)^3(x+1)$ and $4xy(x+1)^2(x-1)$

Now, $16x^3(x-1)^3(x+1)$

$$= 2 \times 2 \times 2 \times 2 \times x \times x \times x \times (x-1) \times (x-1) \times (x-1) \times (x+1)$$

$$\text{and } 4xy(x+1)^2(x-1) = 2 \times 2xy(x+1)(x+1)(x-1)$$

$$\therefore \text{H.C.F.} = 2 \times 2 \times x \times (x+1)(x-1) = 4x(x^2-1)$$

MULTIPLES : Multiples of a number are all those numbers which can be divided completely by the given number.

For example, Multiples of 5 are 5, 10, 15, 20 etc.

Common Multiples : Common multiples of two or more numbers are the numbers which can be exactly divided by each of the given number.

For example, multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 etc. and multiples of 4 are 4, 8, 12, 16, 20, 24, 28 etc.

$\therefore$ Common multiples of 3 and 4 are 12, 24 etc.

Least Common Multiple : The least common multiple (L.C.M.) of two or more numbers is the smallest number which is exactly divisible by each of them.

L.C.M. can be calculated by :

- (i) Prime factorisation method
- (ii) Division method
- (i) **L.C.M. by Prime Factorisation Method :**

Example 4 Find the L.C.M. of 12 and 20 by prime factorisation method.

$$\text{Solution : } 12 = 2 \times 2 \times 3 \text{ and } 20 = 2 \times 2 \times 5$$

$$\therefore \text{L.C.M.} = 2 \times 2 \times 3 \times 5 = 60.$$

- (ii) **L.C.M. by Division Method :**

Example 5 Find the L.C.M. of 14, 56, 91 and 84.

$$\begin{array}{r} \text{Solution : } 14, 56, 91, 84 \\ 2 \overline{)14, 56, 91, 84} \\ \underline{2 7, 28, 91, 42} \\ 7 \overline{)7, 14, 91, 21} \\ \underline{7 0, 14, 21} \\ 1, 2, 13, 3 \end{array}$$

$$\therefore \text{L.C.M.} = 2 \times 2 \times 7 \times 2 \times 13 \times 3 = 2184.$$

LCM of Polynomials : When two or more polynomials are factorised, the product of the factors with highest powers is the lowest common multiple (LCM) of the polynomials.

For example : Consider the polynomials (x^3-8) and (x^2-4)

$$x^3-8 = (x-2)(x^2+2x+4)$$

$$\text{and } x^2-4 = (x+2)(x-2)$$

$$\therefore \text{LCM} = (x-2)(x+2)(x^2+2x+4)$$

$$= (x+2)(x^3-8)$$

H.C.F. AND L.C.M. OF FRACTIONS :

First express the given fractions in their lowest terms. Then,

$$\text{H.C.F.} = \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

$$\text{L.C.M.} = \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

Example 6 Find the H.C.F. and L.C.M. of $4\frac{1}{2}, 6\frac{6}{2}, 10\frac{1}{2}$.

Solution : Here, $4\frac{1}{2} = \frac{9}{2}, 6\frac{6}{2} = 3, 10\frac{1}{2} = \frac{21}{2}$.

$$\text{H.C.F.} = \frac{\text{H.C.F. of } 9, 3, 21}{\text{L.C.M. of } 2, 1, 2} = \frac{3}{2} = 1\frac{1}{2}.$$

$$\text{L.C.M.} = \frac{\text{L.C.M. of } 9, 3, 21}{\text{H.C.F. of } 2, 1, 2} = \frac{63}{1} = 63.$$

H.C.F. AND L.C.M. OF DECIMAL NUMBERS

HCF of Decimal Numbers :

STEP I : Find the HCF of given numbers without decimal.

STEP II : Put the decimal point from right to left according to the maximum decimal places among the given numbers.

LCM of Decimal Numbers :

STEP I : Find the LCM of given numbers without decimal.

STEP II : Put the decimal point from right to left according to the minimum decimal places among the given numbers.

Example 7 Find the H.C.F. and L.C.M. of 0.6, 9.6 and 0.36.

Solution : HCF of 60, 96 and 36 = 12

∴ Required HCF = 0.12

LCM of 60, 96 and 36 = 1440

∴ Required LCM = 144.0

THE PRODUCT OF TWO NUMBERS :

H.C.F. of numbers × L.C.M. of numbers = Product of numbers

Example 8 If H.C.F. and L.C.M. of two numbers are 3 and 60 respectively and one number is 12 then find the other number.

Solution : Let the other number be x .

Product of numbers = H.C.F. × L.C.M.

$$x \times 12 = 3 \times 60$$

$$x = \frac{3 \times 60}{12} = 15.$$

REMEMBER

The greatest number that will exactly divide x, y, z = HCF of x, y and z .

The greatest number that will divide x, y and z leaving remainders a, b and c respectively = HCF of $(x - a), (y - b)$ and $(z - c)$.

The least number which is exactly divisible by x, y and z = LCM of x, y and z .

The least number which when divided by x, y and z leaves the remainder a, b and c respectively = LCM of $(x, y \text{ and } z) - R$ where $R = (x - a) = (y - b) = (z - c)$.

The least number which when divided by x, y and z leaves the same remainder r in each case = LCM of $(x, y \text{ and } z) + r$.

The greatest number that will divide x, y and z leaving the same remainder in each case = HCF of $(x - y), (y - z)$ and $(z - x)$.

To find the n -digit greatest number which, when divided by x, y and z

(a) leaves no remainder (ie. exactly divisible)

⇒ L.C.M. of x, y and $z = L$

$$\Rightarrow \frac{\sqrt[n]{n\text{-digit number}}}{\text{remainder} = R}$$

⇒ Required number = n digit greater number $- R$

(b) Leaves remainder k in each case. then, required number = $(n \text{ digit greatest number} - R) + k$

To find the n -digit smallest number which when divided by x, y and z

(a) Leaves remainder (ie. exactly divisible)

⇒ L.C.M. of x, y and $z = L$

$$\Rightarrow \frac{\sqrt[n]{n\text{-digit smaller number}}}{\text{remainder} = R}$$

⇒ Required number = n digit smallest number $+ (L - R)$

(b) Leaves remainder k in each case

Required number = n digit smallest number $+ (L - R) + k$

Chapter 04: Squares and Square Roots

FUNDAMENTAL OPERATIONS:

1. ADDITION:

- (a) Sum of two positive numbers is a positive number.
For example : $(+5) + (+2) = +7$
- (b) Sum of two negative numbers is a negative number.
For example : $(-5) + (-3) = -8$
- (c) Sum of a positive and a negative number is the difference between their magnitudes and give the sign of the number with greater magnitude.
For example : $(-3) + (+5) = 2$ and $(-7) + (+2) = -5$

2. SUBTRACTIONS :

Subtraction of two numbers is same as the sum of a positive and a negative number.

In subtraction of two negative numbers, sign of second number will change and become positive.

For Example :

$$(+9) - (+2) = (+9) + (-2) = 7$$

$$(-3) - (-5) = (-3) + 5 = +2$$

3. MULTIPLICATION:

- (a) Product of two positive numbers is positive.
- (b) Product of two negative numbers is positive.
- (c) Product of a positive number and a negative number is negative.
- (d) Product of more than two numbers is positive or negative depending upon the presence of negative quantities.

If the number of negative numbers is even then product is positive and if the number of negative numbers is odd then product is negative.

For Example :

$$(-3) \times (+2) = -6$$

$$(-5) \times (-7) = +35$$

$$(-2) \times (-3) \times (-5) = -30$$

$$(-2) \times (-3) \times (+5) = +30$$

4. DIVISION:

- (a) If both the dividend and the divisor are of same sign, then quotient is always positive.
- (b) If the dividend and the divisor are of different sign, then quotient is negative,

For Example :

$$(-36) \div (+9) = -4$$

$$(-35) \div (-7) = +5$$

BRACKETS:

Types of brackets are :

- (i) Vinculum or bar '____'
- (ii) Parenthesis or small or common brackets : ()
- (iii) Curly or middle brackets : { }
- (iv) Square or big brackets : []

The order for removal of brackets is {}, [], ____

If there is a minus (-) sign before the bracket then while removing bracket, sign of each term will change.

'BODMAS' Rule

Now a days it becomes 'VBODMAS' where,

'V' stands for "Vinculum"

'B' stands for "Bracket"

'O' stands for "Of"

'D' stands for "Division"

'M' stands for "Multiplication"

'A' stands for "Addition"

'S' stands for "Subtraction"

Same order of operations must be applied during simplification.

Example 1

$$6 + 5 - 3 \times 2 \text{ of } 5 - (15 \div 7 - 2)$$

$$= 6 + 5 - 3 \times 2 \text{ of } 5 - (15 \div 5) \quad \{\text{Remove vinculum}\}$$

$$= 6 + 5 - 3 \times 2 \text{ of } 5 - 3 \quad \{\text{Remove common bracket}\}$$

$$= 6 + 5 - 3 \times 10 - 3 \quad \{\text{'Of' is done}\}$$

$$= 6 + 5 - 30 - 3 \quad \{\text{Multiplication is done}\}$$

$$= 11 - 33 \quad \{\text{Addition is done}\}$$

$$= -22 \quad \{\text{Subtraction is done}\}$$

Example 2

Simplify : $7 - 2 + 13 - 5 - 2 + 1$

Solution :

$$7 - 2 + 13 - 5 - 2 + 1$$

$$= 7 + 13 + 1 - 2 - 5 - 2 = 21 - 9 = 12$$

$$[7 + 13 + 1 = 21 \text{ and } -2 - 5 - 2 = -9]$$

Example 3

What is the missing figure in the expression given

below?

$$\frac{16}{7} \times \frac{16}{7} - \frac{*}{7} \times \frac{9}{7} + \frac{9}{7} \times \frac{9}{7} = 1$$

Solution :

Let the missing figure in the expression be x.

$$\frac{16}{7} \times \frac{16}{7} - \frac{x}{7} \times \frac{9}{7} + \frac{9}{7} \times \frac{9}{7} = 1$$

$$\Rightarrow 16 \times 16 - 9x + 9 \times 9 = 7 \times 7$$

$$\Rightarrow 9x = 16 \times 16 + 9 \times 9 - 7 \times 7$$

$$= 256 + 81 - 49 = 288$$

$$\Rightarrow x = \frac{288}{9} = 32$$

SQUARE AND SQUARE ROOTS

SQUARE :

When a number is multiplied by itself, we get square of that number.

For example : $4 \times 4 = 16$; we say that the square of 4 is 16.

x^n is also read as x raised to the power n .

Square of a even number is always even.

Square of a odd number is always odd.

Squares of first 30 natural numbers

x	x^2	x	x^2
1	1	16	256
2	4	17	289
3	9	18	324
4	16	19	361
5	25	20	400
6	36	21	441
7	49	22	484
8	64	23	529
9	81	24	576
10	100	25	625
11	121	26	676
12	144	27	729
13	169	28	784
14	196	29	841
15	225	30	900

Perfect Square :

A natural number is called a perfect square, if it is the square of some natural number.

Numbers 1, 4, 9, 16, 25, 36, etc. all are perfect squares.

To find out whether the given number is perfect square :

Express the number as a product of prime factors.

If it is expressible as the product of pairs of equal factors, then it is a perfect square.

Example 4 Is 144 a perfect square ?

Solution :

$$\begin{array}{r} 2 \overline{)144} \\ 2 \overline{)72} \\ 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \overline{)3} \\ 1 \end{array}$$

$$\therefore 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

Since both the factors in each group are equal hence it is a perfect square.

REMEMBER

A number ending in an odd number of zeros is never a perfect square. For example, 15000.

A number ending in 2, 3, 7 or 8 is never a perfect square.

The square of a natural number (other than 1) is a multiple of 3 or exceeds a multiple of 3 by 1.

$$\text{For example : } 2^2 = 4 = (3 \times 1) + 1$$

$$3^2 = 9 = (3 \times 3)$$

$$4^2 = 16 = (3 \times 5) + 1$$

The square of a natural number (other than 1) is a multiple of 4 or exceeds a multiple of 4 by 1.

$$\text{For example : } 2^2 = 4 = 4 \times 1$$

$$3^2 = 9 = (4 \times 2) + 1$$

$$4^2 = 16 = (4 \times 4)$$

$$5^2 = 25 = (4 \times 6) + 1.$$

For a natural number m (other than 1) $2m$, $(m^2 - 1)$ and $(m^2 + 1)$ are pythagorean triplets.

For example : take $m = 4$ then,

$$2m = 2 \times 4 = 8$$

$$m^2 - 1 = 4^2 - 1 = 15$$

$$m^2 + 1 = 4^2 + 1 = 17$$

$$8^2 + 15^2 = 64 + 225 = 289 = 17^2$$

So, 8, 15 and 17 are pythagorean triplets.

Alternative method to find the square of a number :

STEP I: Express the given number as a sum or difference of two numbers.

STEP II: Apply the one of the following formule

$$(a + b)^2 = a^2 + 2ab + b^2; \quad (a - b)^2 = a^2 - 2ab + b^2$$

Example 5 Find the square of 151.

Solution :

$$(151)^2 = (150 + 1)^2$$

$$\text{Here } a = 150, b = 1$$

$$\begin{aligned} \therefore (151)^2 &= (150)^2 + 2 \times 150 \times 1 + (1)^2 \\ &= 22500 + 300 + 1 \\ &= 22801 \end{aligned}$$

Example 6 Find the square of 679.

Solution :

$$\begin{aligned} (679)^2 &= (700 - 21)^2 \\ &= 490000 - 2 \times 700 \times 21 + 441 \\ &= 461041 \end{aligned}$$

SQUARE ROOTS :

The square root of a number 'x' is that number which when multiplied by itself gives 'x' as the product.

Square root of x is denoted by the symbol $\sqrt{x}$

For Example :

$$(i) \quad \text{Square root of 4 is 2 or } \sqrt{4} = 2$$

$$\begin{aligned} (ii) \quad \sqrt{2.56} &= \sqrt{\frac{256}{100}} = \sqrt{\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{10 \times 10}} \\ &= \frac{2 \times 2 \times 2 \times 2}{10} = \frac{16}{10} = 1.6 \end{aligned}$$

Whether the number is negative or positive, its square is always positive and so, the square root of a negative number is not possible.

Square roots of first 20 natural numbers

x	$\sqrt{x}$	x	$\sqrt{x}$
1	1.000	11	3.317
2	1.414	12	3.464
3	1.732	13	3.606
4	2.000	14	3.742
5	2.236	15	3.873
6	2.449	16	4.000
7	2.646	17	4.123
8	2.828	18	4.243
9	3.000	19	4.359
10	3.162	20	4.472

Finding the square root of a perfect square number by Prime Factorisation Method :

1. Resolve the given number into prime factors.
2. Make pairs of similar factors.
3. Take the product of prime factors choosing one out of every pair.

Example 7 Find the square root of 1521.

Solution :

$$\begin{array}{r} 3 \overline{)1521} \\ 3 \overline{)507} \\ 13 \overline{)169} \\ 13 \overline{)13} \\ 1 \end{array}$$

$$1521 = 3 \times 3 \times 13 \times 13$$

$$\sqrt{1521} = 3 \times 13 = 39$$

Example 8 Find the smallest number by which 396 must be multiplied so that the product becomes a perfect square.

Solution :

By Prime factorisation, we get

$$\begin{array}{r} 2 \overline{)396} \\ 2 \overline{)198} \\ 3 \overline{)99} \\ 3 \overline{)33} \\ 11 \overline{)11} \\ 1 \end{array}$$

$$396 = 2 \times 2 \times 3 \times 3 \times 11$$

In order to become a perfect square, one more 11 is required.

Example 9 Find the smallest number by which 6300 be divided, so that the quotient is a perfect square.

Solution :

$$\begin{array}{r} 2 \overline{)6300} \\ 2 \overline{)3150} \\ 3 \overline{)1575} \\ 3 \overline{)525} \\ 5 \overline{)175} \\ 5 \overline{)35} \\ 7 \overline{)7} \\ 1 \end{array}$$

$$6300 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$$

Since, the prime factor 7 is not paired.

$\therefore$ The given number should be divided by 7.

Finding the square root by Long Division Method :

1. Mark off the digits in pairs starting from right to left.

2. Find the largest or think of the largest whole number which when multiplied by itself is equal or nearest to the left most pair. The number is taken as the divisor as well as quotient.
3. Subtract the product from left most pair. There will be a remainder, bring down the next pair of digits.
4. Now, for next divisor add the quotient to previous divisor. Go on repeating the above steps till all the pairs have been taken up. The quotient so obtained is the required squareroot of the given number.

Example 10 Find the square root of 106276.

Solution : By long division method :

$$\begin{array}{r} 326 \\ 3 \overline{)106276} \\ 9 \\ \hline 62 \\ 62 \\ \hline 646 \\ 3876 \\ \hline 3876 \\ 3876 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{106276} = 326$$

Example 11 What least number must be subtracted from 46687 to get a perfect square ? Also, find the square root of this perfect square.

Solution :

Let's find the square root of 46687.

$$\begin{array}{r} 216 \\ 2 \overline{)46687} \\ 4 \\ \hline 41 \\ 66 \\ \hline 41 \\ 2587 \\ \hline 426 \\ 2556 \\ \hline 31 \end{array}$$

This shows that $(216)^2$ is less than 46687 by 31. So, in order to get a perfect square, 31 must be subtracted from the given number.

$$\therefore \text{Required perfect square number} = 46687 - 31 = 46656$$

$$\text{Also, } \sqrt{46656} = 216$$

Example 12 Find the least number of six digits which is a perfect square. Also, find the square root of the number.

Solution : The least number of six digits = 100000

Square root of 100000 :

$$\begin{array}{r} 316 \\ 3 \overline{)100000} \\ 9 \\ \hline 61 \\ 100 \\ \hline 61 \\ 3900 \\ \hline 626 \\ 3756 \\ \hline 144 \end{array}$$

$$\text{Hence, } (316)^2 < 100000 < (317)^2$$

$$\therefore \text{The least number to be added} = (317)^2 - 100000 = 489$$

$$\text{Hence, the required number} = 100000 + 489 = 100489$$

$$\therefore \sqrt{100489} = 317$$

Chapter 05: Cube and Cube Roots, Surds and Indices

CUBE AND CUBE ROOTS

CUBE:

Cube of a number is that number whose raised to the power 3.

For example :

$$4^3 = 4 \times 4 \times 4 = 64; \text{ we say the cube of 4 is 64.}$$

Cubes of First 20 Natural Numbers

x	x^3	x	x^3
1	1	11	1331
2	8	12	1728
3	27	13	2197
4	64	14	2744
5	125	15	3375
6	216	16	4096
7	343	17	4913
8	512	18	5832
9	729	19	6859
10	1000	20	8000

Perfect Cube : A natural number is said to be a perfect cube if it is the cube of some natural number.

If m is a natural number, then m^3 is a perfect cube. For example, $10^3 = 1000$ is a perfect cube.

Example 18 Is 343 a perfect cube?

Solution :

$$\begin{array}{r} 7 \overline{)343} \\ 7 \overline{)49} \\ 7 \overline{)7} \\ 1 \end{array}$$

$$\therefore 343 = 7 \times 7 \times 7$$

Hence, it is a perfect cube.

- Cubes of all even numbers are even.
- Cubes of all odd numbers are odd.
- Cubes of negative integers are negative.

Finding the cube of a number : To find the cube of given number, multiply the number with itself three times.

For example : $8^3 = 8 \times 8 \times 8 = 512$

Alternative method to find the cube of a number :

STEP I : Express the given number as a sum or difference of two numbers.

STEP II : Apply the one of the following formula :

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Example 19 Find the cube of 33.

Solution :

$$(33)^3 = (30 + 3)^3$$

$$\text{Here } a = 30, b = 3$$

$$\begin{aligned} \therefore (33)^3 &= 27000 + 3 \times (30)^2 \times 3 + 3(30)(3)^2 + 27 \\ &= 27000 + 8100 + 810 + 27 \\ &= 35937 \end{aligned}$$

Cube Root : The cube root of a number x is that number whose cube gives x . The cube root of x is denoted by the symbol $\sqrt[3]{x}$.

Cube Roots of Some Natural Numbers

x	$\sqrt[3]{x}$	x	$\sqrt[3]{x}$
1	1.000	20	2.714
2	1.260	30	3.107
3	1.442	40	3.420
4	1.587	50	3.684
5	1.710	60	3.915
6	1.817	70	4.121
7	1.913	80	4.309
8	2.000	90	4.481
9	2.080	100	4.642
10	2.154		

If cube ends in 1, then its cube roots end in 1.

REMEMBER

Unit digit of cube	1	2	3	4	5	6	7	8	9	0
Unit (or extreme right) digit of cube root	1	8	7	4	5	6	3	2	9	0

Hence, 1, 4, 5, 6, 9 and 0 repeat themselves and 2, 3, 7 and 8 complement of 10 i.e. 8, 7, 3 and 2 respectively.

Finding the cube root of a perfect cube by prime factorisation method :

1. Resolve the given number into prime factors.
2. Make triplet of similar factors.
3. Take the product of prime factors choosing one out of every triplet number.
4. The product is the required cube root of the given number.

Example 20 Find the cube root of 2744.

Solution :

$$\begin{array}{r} 2 \overline{)2744} \\ 2 \overline{)1372} \\ 2 \overline{)686} \\ 7 \overline{)343} \\ 7 \overline{)49} \\ 7 \overline{)7} \\ 1 \end{array}$$

$$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

$$\therefore \sqrt[3]{2744} = 2 \times 7 = 14$$

Alternative method to find the cube roots of exact cubes having upto 6 digits :

STEP I : Form the groups of three consecutive digits, starting from the extreme right. The last group may consists of less than three digits. The number of such 3-digit groups equals the number of digits in the cube root.

If the number consists of upto 3 digits, its cube root will be less than 10 and can be easily found from the table.

For number consisting of 4, 5 or 6 digits, the cube root will consist of 2 digits and its right most digit can be found by the table.

STEP II: Find the largest number whose cube is less than or equal to the number under the left most bar. This is the left most digit (L) of cube root.

Example 21 Find the cube root of 9261

Solution :

We have : $\overline{9\ 261}$

$\therefore$ 9 lies between 2^3 and 3^3

So L = 2

Now, 261 ends in 1. So R = 1

$\therefore$ number is 21.

Cube Root of a Negative Perfect Cube :

If a is a positive integer then $-a$ is a negative integer.

$$(-a)^3 = -a^3$$

$$\text{So, } \sqrt[3]{(-a^3)} = -a$$

In general, we have $\sqrt[3]{-x} = -\sqrt[3]{x}$.

Example 22 Find the cube root of -74088 .

Solution :

$$74088 = \overline{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7}$$

$$\text{So, } \sqrt[3]{74088} = 2 \times 3 \times 7 = 42$$

$$\text{Hence } \sqrt[3]{-74088} = -\sqrt[3]{74088} = -42$$

REMEMBER

For any two integers a and b , we have

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \quad \text{and} \quad \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

Example 23 Find the cube root of 2.744

Solution :

$$\sqrt[3]{2.744} = \sqrt[3]{\frac{2744}{1000}} = \frac{\sqrt[3]{2744}}{\sqrt[3]{1000}}$$

$$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

$$\therefore \sqrt[3]{2744} = 2 \times 7 = 14$$

$$\text{Also } \sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10} = 10$$

$$\text{So, } \frac{\sqrt[3]{2744}}{\sqrt[3]{1000}} = \frac{14}{10} = 1.4$$

Example 24 Find the cube root of 658503.

Solution :

$$\begin{aligned} \text{(c) } \sqrt[3]{658503} &= 219501 \times 3 \\ &= 73167 \times 3 \times 3 \\ &= 24389 \times 3 \times 3 \times 3 \\ &= 841 \times 29 \times 3 \times 3 \times 3 \\ &= 29 \times 29 \times 29 \times 3 \times 3 \times 3 \\ &= 29 \times 3 = 87 \end{aligned}$$

SURDS AND INDICES

INDICES

When a number ' a ' is multiplied by itself ' m ' times, then we say that ' a is of m -indices'. a^m is read as ' a raised to the power m '.

Laws of Indices :

$$\text{I} \quad a^m \times a^n = a^{m+n}$$

Example 25 Simplify : $(1000)^7 \times (10)^5$

Solution :

$$(10^3)^7 \times (10)^5 = 10^{21+5} = 10^{26}$$

$$\text{II} \quad \frac{a^m}{a^n} = a^{m-n}$$

Example 26 Simplify : $(5)^{25} \div (125)^8$

Solution :

$$5^{25} \div (5^3)^8 = 5^{25} \div 5^{24} = 5^{25-24} = 5$$

$$\text{III} \quad (a^m)^n = a^{mn}$$

Example 27 Simplify : $\left[(\sqrt{2})^3\right]^4$

Solution :

$$\left[(\sqrt{2})^3\right]^4 = (2)^{\frac{3}{2} \times 4} = 2^6$$

$$\text{IV} \quad (ab)^n = a^n b^n$$

$$\text{For Example : } (12)^2 = (4 \times 3)^2 = 4^2 \times 3^2 = 16 \times 9 = 144$$

$$\text{V} \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\text{For Example : } \left(\frac{3}{4}\right)^4 = \frac{3^4}{4^4} = \frac{81}{256}$$

$$\text{VI} \quad a^0 = 1$$

$$\text{For Example : } 4^0 = 1$$

REMEMBER

$$a^m + a^n \neq a^{m+n}$$

$$a^m \times b^n \neq (ab)^{m+n}$$

SURDS :

If ' a ' is a rational number and ' n ' is a positive integer such that the n th root of a i.e. $a^{1/n}$ or $\sqrt[n]{a}$ is an irrational number, then $a^{1/n}$ is called a surd or radical of order n and a is called the radicand.

For Example : $\sqrt{2}$, $\sqrt[3]{3}$, etc.

Consider the real number $\sqrt{2+\sqrt{3}}$. Since $2+\sqrt{3}$ is not a rational number, therefore, $\sqrt{2+\sqrt{3}}$ is not a surd.

Mixed Surds : A rational factor and a surd multiplied together are called mixed surds.

For Example : $3\sqrt{2}$, $5\sqrt[3]{6}$ etc.

Pure Surd : A surd which has unity only as rational factor, the other factor being irrational, is called a pure surd.

For Example : $\sqrt{3}$, $\sqrt[3]{2}$, $\sqrt[4]{7}$.

Quadratic Surd : A surd of order 2 is called a quadratic surd.

For Example : $\sqrt{7} = 7^{1/2}$ and $\sqrt{13} = 13^{1/2}$

Cubic Surd : A surd of order 3 is called a cubic surd.

For Example : $\sqrt[3]{4} = (4)^{1/3}$.

Biquadratic Surd : A surd of order 4 is called a biquadratic surd. A biquadratic surd is also called **quartic surd**.

For Example : $\sqrt[4]{5}$ is a biquadratic surd but $\sqrt[4]{81} (= 3)$ is not a biquadratic surd as it is not a surd.

Laws of Surds :

I $(\sqrt[n]{a})^n = a$

Example 28 Simplify : (i) $(\sqrt[3]{7})^3$ (ii) $\sqrt[3]{27}$

Solution :

(i) $(\sqrt[3]{7})^3 = [(7)^{1/3}]^3 = 7^{1/3 \times 3} = 7$

(ii) $\sqrt[3]{27} = \sqrt[3]{3^3} = (3^3)^{1/3} = 3$

Example 29 Solve : $\sqrt[4]{3x+1} = 2$

Solution :

$$\sqrt[4]{3x+1} = 2$$

$$\Rightarrow (\sqrt[4]{3x+1})^4 = (2)^4$$

$$\Rightarrow 3x+1 = 16$$

$$\Rightarrow 3x = 15$$

$$\therefore x = 5$$

II $\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab}$

Example 30 Simplify : $\sqrt[3]{3} \cdot \sqrt[3]{4}$

Solution :

$$\sqrt[3]{3} \cdot \sqrt[3]{4} = \sqrt[3]{3 \cdot 4} = \sqrt[3]{12}$$

III $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Example 31 Find $\sqrt[3]{\frac{125}{64}}$

Solution :

$$\sqrt[3]{\frac{125}{64}} = \frac{\sqrt[3]{125}}{\sqrt[3]{64}} = \frac{\sqrt[3]{5^3}}{\sqrt[3]{4^3}} = \frac{5}{4}$$

IV $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

Example 32 Simplify : $\sqrt[3]{\sqrt[4]{5}}$

Solution :

$$\sqrt[3]{\sqrt[4]{5}} = \sqrt[12]{5}$$

Conversion of Mixed Surds into Pure Surds :

Example 33 Express each of the following as a pure surd :

(i) $2\sqrt[3]{4}$ (ii) $\frac{2}{3}\sqrt[3]{108}$

Solution :

(i) $2\sqrt[3]{4} = 2 \times 4^{1/3} = (2^3)^{1/3} \times 4^{1/3}$
 $= (8)^{1/3} \times 4^{1/3} = (8 \times 4)^{1/3} = (32)^{1/3} = \sqrt[3]{32}$

(ii) $\frac{2}{3}\sqrt[3]{108} = \frac{2}{3} \times (108)^{1/3}$
 $= \left[\left(\frac{2}{3} \right)^3 \right]^{1/3} \times (108)^{1/3}$
 $= \left(\frac{8}{27} \right)^{1/3} \times (108)^{1/3} = \left(\frac{8}{27} \times 108 \right)^{1/3}$
 $= (8 \times 4)^{1/3} = (32)^{1/3} = \sqrt[3]{32}$

Conversion of Surds into Surds of the Same Order :

Let the surds be $\sqrt[n_1]{a_1}$, $\sqrt[n_2]{a_2}$, $\sqrt[n_3]{a_3}$, ...

STEP I : Compute L.C.M. of $n_1, n_2, n_3, \dots$

Let L.C.M. = n

STEP II : Compute $\frac{n}{n_1}, \frac{n}{n_2}, \frac{n}{n_3}, \dots$

$= m_1, m_2, m_3, \dots$

STEP III : Required surds are

$$\sqrt[n]{a_1^{m_1}}, \sqrt[n]{a_2^{m_2}}, \sqrt[n]{a_3^{m_3}}, \dots$$

Example 34 Express $\sqrt[4]{2}$, $\sqrt[3]{3}$, $\sqrt[5]{4^2}$ as surds of the same order.

Solution :

Here, $n_1 = 4$, $n_2 = 3$, $n_3 = 5$

$\therefore n = \text{L.C.M. of } (4, 3, 5) = 60$

$$m_1 = \frac{n}{n_1} = \frac{60}{4} = 15, m_2 = \frac{60}{3} = 20, m_3 = \frac{60}{5} = 12$$

$$\therefore \sqrt[4]{2} = \sqrt[60]{2^{15}}; \sqrt[3]{3} = \sqrt[60]{3^{20}}$$

$$\sqrt[5]{4^2} = \sqrt[60]{(4^2)^{12}} = \sqrt[60]{4^{24}}$$

Comparison of Surds :

STEP I : Convert each surd into a surd of same order.

STEP II : Compare the radicand of the surds. The surd with larger radicand is the largest of the given surds.

Chapter 06: Clocks and Calender

CLOCK

INTRODUCTION

A clock has two hands : Hour hand and Minute hand.

The minute hand (M.H.) is also called the long hand and the hour hand (H.H.) is also called the short hand.

The clock has 12 hours numbered from 1 to 12.

Also, the clock is divided into 60 equal minute divisions. Therefore, each hour number is separated by five minute divisions. Therefore,

One minute division = $\frac{360}{60} = 6^\circ$ apart. i.e. In one minute, the minute hand moves 6° .

One hour division = $6^\circ \times 5 = 30^\circ$ apart. i.e., in one hour, the hour hand moves 30° apart.

Also, in one minute, the hour hand moves = $\frac{30^\circ}{60} = \frac{1^\circ}{2}$ apart.

Since, in one minute, minute hand moves 6° and hour hand moves $\frac{1^\circ}{2}$, therefore, in one minute, the minute hand gains

$5\frac{1}{2}^\circ$ more than hour hand.

In one hour, the minute hand gains $5\frac{1}{2} \times 60 = 330^\circ$ over the hour hand. i.e., the minute hand gains 55 minutes divisions over the hour hand.

Relative Position of the Hands.

The position of the M.H. relative to the H.H. is said to be the same, whenever the M.H. is separated from the H.H. by the same number of minute divisions and is on same side (clockwise or anticlockwise) of the H.H.

Any relative position of the hands of a clock is repeated 11 times in every 12 hours.

- When both hands are 15 minute spaces apart, they are at right angle.
- When they are 30 minute spaces apart, they point in opposite directions.
- The hands are in the same straight line when they are coincident or opposite to each other.

In every hour, both the hand coincide once.

In a day, the hands are coinciding 22 times.

In every 12 hours, the hands of clock coincide 11 times.

In every 12 hours, the hands of clock are in opposite direction 11 times.

In every 12 hours, the hands of clock are at right angles 22 times.

In every hour, the two hands are at right angles 2 times.

In every hour, the two hands are in opposite direction once.

In a day, the two hands are at right angles 44 times.

If both the hands coincide, then they will again coincide

after $65\frac{5}{11}$ minutes. i.e. in correct clock, both hand

coincide at an interval of $65\frac{5}{11}$ minutes.

If the two hands coincide in time less than $65\frac{5}{11}$ minutes, then clock is too fast and if the two hands

coincides in time more than $65\frac{5}{11}$ minutes, then the

clock is too slow.

Another one shortcut formula for clocks

$$\text{Angle} = \left| 30H - \left(\frac{11}{2} \right) M \right|;$$

H $\Rightarrow$ hour

M $\Rightarrow$ minutes.

Example 1 At what time between 4 and 5 will the hands of a watch

- coincide, and
- point in opposite directions.

Solution : (i) At 4 O'clock, the hands are 20 minutes apart. Clearly the minute hand must gain 20 minutes before two hands can be coincident.

But the minute-hand gains 55 minutes in 60 minutes.

Let minute hand will gain x minute in 20 minutes.

$$\text{So, } \frac{55}{60} = \frac{x}{20}$$

$$\Rightarrow x = \frac{20 \times 60}{55} = \frac{240}{11} = 21\frac{9}{11} \text{ min.}$$

$\therefore$ The hands will be together at $21\frac{9}{11}$ min past 4.

- Hands will be opposite to each other when there is a space of 30 minutes between them. This will happen when the minute hand gains $(20 + 30) = 50$ minutes.

Now, the minute hand gains 50 min in $\frac{50 \times 60}{55}$ or $54\frac{6}{11}$ min.

$\therefore$ The hands are opposite to each other at $54\frac{6}{11}$ min past 4.

Example 2 What is the angle between the hour hand and minute hand when it was 5 : 05 pm.

Solution : 5.05 pm means hour hand was on 5 and minute hand was on 1, i.e., there will be 20 minutes gap.

$$\therefore \text{Angle} = 20 \times 6^\circ = 120^\circ \quad [\because 1 \text{ minute} = 6^\circ]$$

INCORRECT CLOCK

If a clock indicates 6 : 10, when the correct time is 6 : 00, it is said to be 10 minute too fast and if it indicates 5 : 50 when the correct time is 6 : 00, it is said to be 10 minute too slow.

Also, if both hands coincide at an interval x minutes and

$$x < 65\frac{5}{11}, \text{ then total time gained} = \left(\frac{65\frac{5}{11} - x}{x} \right) \text{ minutes}$$

and clock is said to be 'fast'.

If both hands coincide at an interval x minutes and

$$x > 65\frac{5}{11}, \text{ then total time lost} = \left(\frac{x - 65\frac{5}{11}}{x} \right) \text{ minutes and}$$

clock is said to be 'slow'.

Example 3 My watch, which gains uniformly, is 2 min slow at noon on Sunday, and is 4 minutes 48 seconds fast at 2 pm on the following Sunday. When was it correct.

Solution : From Sunday noon to the following Sunday at 2 pm = 7 days 2 hours = 170 hours.

The watch gains $\left(2 + 4\frac{48}{60} \right) = 6\frac{4}{5}$ minutes in 170 hours.

$$\therefore \text{The watch gains 2 minutes in } \frac{2}{6\frac{4}{5}} \times 170 = 50 \text{ hours}$$

Now, 50 hours = 2 days 2 hours

2 days 2 hours from Sunday noon = 2 pm on Tuesday.

Example 4 The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose ?

Solution : In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand.

$$55 \text{ min. are gained in } \left(\frac{60}{55} \times 60 \right) \text{ min.} = 65\frac{5}{11} \text{ min.}$$

But, they are together after 65 min.

$$\therefore \text{Gain in 65 min.} = \left(65\frac{5}{11} - 65 \right) = \frac{5}{11} \text{ min.}$$

$$\text{Gain in 24 hours} = \left(\frac{5}{11} \times \frac{60 \times 24}{65} \right) \text{ min.} = 10\frac{10}{143} \text{ min.}$$

$$\therefore \text{The clock gains } 10\frac{10}{143} \text{ minutes in 24 hours.}$$

Example 5 A man who went out between 5 or 6 and returned between 6 and 7 found that the hands of the watch had exactly changed place. When did he go out ?

Solution : Between 5 and 6 to 6 and 7, hands will change place after crossing each other one time. i.e., they together will make $1 + 1 = 2$ complete revolutions.

H.H. will move through $2 \times \frac{60}{13}$ or $\frac{120}{13}$ minute divisions.

Between 5 and 6 $\rightarrow \frac{120}{13}$ minute divisions.

At 5, minute hand is 25 minute divisions behind the hour-hand.

Hence it will have to gain $25 + \frac{120}{13}$ minute divisions on

the hour-hand = $\frac{445}{13}$ minute divisions on the hour hand.

The minute hand gains $\frac{445}{13}$ minute divisions in

$$\frac{445}{13} \times \frac{12}{11} \text{ minutes} = \frac{5340}{143} = 37\frac{49}{143} \text{ minutes}$$

$$\therefore \text{The required time of departure is } 37\frac{49}{143} \text{ minutes past 5.}$$

CALENDER

INTRODUCTION

The solar year consists of 365 days, 5 hrs 48 minutes, 48 seconds.

In 47 BC, Julius Caesar arranged a calendar known as the Julian

calendar in which a year was taken as $365\frac{1}{4}$ days and in order to get rid of the odd quarter of a day, an extra day was added once in every fourth year and this was called as leap year or Bissextile.

Now a days, the calendar, which is mostly used, is arranged by Pope Gregory XII and known as Gregorian calendar.

In India, number of calendars were being used till recently. In 1952, the Government adopted the National Calendar based on Saka era with Chaitra as its first month. In an ordinary year, Chaitra 1 falling on March 22 of Gregorian Calendar and in a leap year it falls on March 21.

REMEMBER

In an ordinary year,

1 year = 365 days = 52 weeks + 1 day

In a leap year,

1 year = 366 days = 52 weeks + 2 days

NOTE First January 1 A.D. was Monday.

100 years or one century contains 76 ordinary years and 24 leap years.

$$\begin{aligned}\Rightarrow & [76 \times 52 \text{ weeks} + 76 \text{ odd days}] + \\ & [24 \times 52 \text{ weeks} + 24 \times 2 \text{ odd days}] \\ = & (76 + 24) \times 52 \text{ weeks} + (76 + 48) \text{ odd days} \\ = & 100 \times 52 \text{ weeks} + 124 \text{ odd days} \\ = & 100 \times 52 \text{ weeks} + (17 \times 7 + 5) \text{ odd days} \\ = & (100 \times 52 + 17) \text{ weeks} + 5 \text{ odd days} \\ \therefore & 100 \text{ years contain 5 odd days.}\end{aligned}$$

Similarly, 200 years contain 3 odd days,
300 years contain 1 odd day,
400 years contain 0 odd days.

Year whose non-zero numbers are multiple of 4 contains no odd days; like 800, 1200, 1600 etc.

The number of odd days in months

The month with 31 days contains $(4 \times 7 + 3)$ i.e., 3 odd days and the month with 30 days contains $(4 \times 7 + 2)$ i.e., 2 odd days.

February in an ordinary year gives no odd days, but in a leap year gives one odd day.

Example 6 What day of the week was 15th August 1949?

Solution : 15th August 1949 means

1948 complete years + first 7 months of the year 1949
+ 15 days of August.

1600 years give no odd days.

300 years give 1 odd day.

48 years give $\{48 + 12\} = 60 = 4$ odd days.

[$\because$ For ordinary years: 48 odd days and for leap year 1 more odd day $(48 \div 4) = 12$ odd days; $60 = 7 \times 8 + 4$]

From 1st January to 15th August 1949

Number of odd days :

January-3; February-0; March-3; April-2; May-3;
June-2; July-3 and August-1

Total 17 i.e., 3 odd days

$\therefore$ 15th August 1949 : $1 + 4 + 3 = 8 = 1$ odd day.

This means that 15th Aug. fell on 1st day. Therefore, the required day was Monday.

Example 7 How many times does the 29th day of the month occur in 400 consecutive years?

Solution : In 400 consecutive years, there are 97 leap years. Hence, in 400 consecutive years, February has the 29th day 97 times and the remaining eleven months have the 29th day $400 \times 11 = 4400$ times

$\therefore$ The 29th day of the month occurs $(4400 + 97)$ or 4497 times.

Example 8 Today is 5th February. The day of the week is Tuesday. This is a leap year. What will be the day of the week on this date after 5 years?

Solution : This is a leap year. So, next 3 years will give one odd day each. Then leap year gives 2 odd days and then again next year gives 1 odd day.

Therefore $(3 + 2 + 1) = 6$ odd days will be there.

Hence the day of the week will be 6 odd days beyond Tuesday, i.e., it will be Monday.

Example 9 Prove that the calendar for 1990 will same for 2001 also.

Solution : It is clear that the calendar for 1990 will same for 2001 if first January of both the years is the same weekdays. For that the number of odd days between 31st December 1989 and 31st December 2000 must be zero. Odd days are as given below.

Year	1990	1991	1992	1993	1994
Odd days	1	1	(Leap) 2	1	1
1995	1996	1997	1998	1999	2000
1	(Leap) 2	1	1	1	(Leap) 2

Total number of odd days = 14 days = 2 weeks + 0 odd days.

Chapter 07: Algebra

VARIABLE :

The unknown quantities used in any equation are known as variables. Generally, they are denoted by the last English alphabet x, y, z etc.

An equation is a statement of equality of two algebraic expressions, which involve one or more unknown quantities, called the variables.

LINEAR EQUATION

An equation in which the highest power of variables is one, is called a linear equation. These equations are called linear because the graph of such equations on the x - y cartesian plane is a straight line.

Linear Equation in One Variable : A linear equation which contains only one variable is called linear equation in one variable.

The general form of such equations is $ax + b = c$, where a, b and c are constants and $a \neq 0$.

All the values of x which satisfy this equation are called its solution(s).

An equation satisfied by all values of the variable is called an identity. For example : $2x + x = 3x$.

Example 1 Solve : $2x - 5 = 1$

Solution : $2x - 5 = 1$

$$\Rightarrow 2x = 1 + 5$$

$$\Rightarrow 2x = 6 \Rightarrow x = \frac{6}{2} = 3.$$

Example 2 Solve : $7x - 5 = 4x + 11$

Solution : $7x - 5 = 4x + 11$

$$\Rightarrow 7x - 4x = 11 + 5 \text{ (Bringing like terms together)}$$

$$\Rightarrow 3x = 16 \Rightarrow x = \frac{16}{3} = 5\frac{1}{3}.$$

Example 3 Solve : $\frac{4}{x} - \frac{3}{2x} = 5$

Solution : $\frac{4}{x} - \frac{3}{2x} = 5 \Rightarrow \frac{8-3}{2x} = 5$

$$\Rightarrow \frac{5}{2x} = 5 \Rightarrow 10x = 5$$

$$\Rightarrow x = \frac{5}{10} = \frac{1}{2}$$

Applications of Linear Equations with One Variables

Example 4 The sum of the digits of a two-digit number is 16. If the number formed by reversing the digits is less than the original number by 18. Find the original number.

Solution : Let unit digit be x .

Then tens digit = $16 - x$

$$\therefore \text{Original number} = 10 \times (16 - x) + x \\ = 160 - 9x.$$

On reversing the digits, we have x at the tens place and $(16 - x)$ at the unit place.

$$\therefore \text{New number} = 10x + (16 - x) = 9x + 16$$

$$\text{Original number} - \text{New number} = 18$$

$$(160 - 9x) - (9x + 16) = 18$$

$$160 - 18x - 16 = 18$$

$$-18x + 144 = 18$$

$$-18x = 18 - 144 \Rightarrow 18x = 126$$

$$\Rightarrow x = 7$$

$\therefore$ In the original number, we have unit digit = 7

$$\text{Tens digit} = (16 - 7) = 9$$

Thus, original number = 97

Example 5 The denominator of a rational number is greater than its numerator by 4. If 4 is subtracted from the numerator and 2 is added to its denominator, the new number

becomes $\frac{1}{6}$. Find the original number.

Solution : Let the numerator be x .

Then, denominator = $x + 4$

$$\therefore \frac{x-4}{x+4+2} = \frac{1}{6}$$

$$\Rightarrow \frac{x-4}{x+6} = \frac{1}{6}$$

$$\Rightarrow 6(x-4) = x+6$$

$$\Rightarrow 6x - 24 = x + 6 \Rightarrow 5x = 30$$

$$\therefore x = 6$$

Thus, Numerator = 6, Denominator = $6 + 4 = 10$.

Hence the original number = $\frac{6}{10}$.

Example 6 A man covers a distance of 33 km in $3\frac{1}{2}$ hours; partly on foot at the rate of 4 km/hr and partly on bicycle at the rate of 10 km/hr. Find the distance covered on foot.

Solution : Let the distance covered on foot be x km.

$$\therefore \text{Distance covered on bicycle} = (33 - x) \text{ km}$$

$$\therefore \text{Time taken on foot} = \frac{\text{Distance}}{\text{Speed}} = \frac{x}{4} \text{ hr.}$$

$$\therefore \text{Time taken on bicycle} = \frac{33-x}{10} \text{ hr.}$$

The total time taken = $\frac{7}{2}$ hr.

$$\frac{x}{4} + \frac{33-x}{10} = \frac{7}{2}$$

$$\frac{5x+66-2x}{20} = \frac{7}{2}$$

$$6x+132=140$$

$$6x=140-132$$

$$6x=8$$

$$x = \frac{8}{6} = 1.33 \text{ km}$$

∴ The distance covered on foot is 1.33 km.

Linear Equation in Two Variables : General equation of a linear equation in two variables is $ax + by + c = 0$, where $a, b \neq 0$ and c is a constant, and x and y are the two variables.

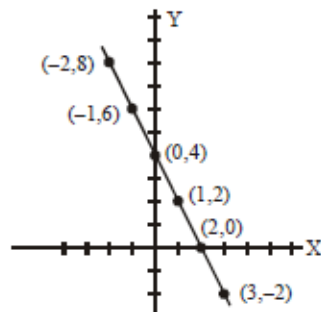
The sets of values of x and y satisfying any equation are called its solution(s).

Consider the equation $2x + y = 4$. Now, if we substitute $x = -2$ in the equation, we obtain $2 \times (-2) + y = 4$ or $-4 + y = 4$ or $y = 8$. Hence $(-2, 8)$ is a solution. If we substitute $x = 3$ in the equation, we obtain $2 \times 3 + y = 4$ or $6 + y = 4$ or $y = -2$.

Hence $(3, -2)$ is a solution. The following table lists six possible values for x and the corresponding values for y , i.e. six solutions of the equation.

x	-2	-1	0	1	2	3
y	8	6	4	2	0	-2

If we plot the solutions of the equation $2x + y = 4$ which appear in the above table then we see that they all lie on the same line. We call this line the graph of the equation since it corresponds precisely to the solution set of the equation.



Example 7 Find the values of x and y which satisfy the equations :

$$4x + 3y = 25 \text{ and } x + 5y = 19.$$

Solution : Substitution Method :

$$4x + 3y = 25 \quad \dots(i)$$

$$x + 5y = 19 \quad \dots(ii)$$

$$\Rightarrow x = 19 - 5y$$

Substitute $x = 19 - 5y$ in equation (i), we get

$$4(19 - 5y) + 3y = 25$$

$$\Rightarrow 76 - 20y + 3y = 25 \Rightarrow 76 - 17y = 25$$

$$\Rightarrow 17y = 51 \Rightarrow y = 3$$

Putting $y = 3$ in equation (ii), we obtain

$$x + 5 \times 3 = 19$$

$$\Rightarrow x + 15 = 19 \Rightarrow x = 4$$

∴ $x = 4$ and $y = 3$ is the solution.

Elimination Method :

$$4x + 3y = 25 \quad \dots(i)$$

$$x + 5y = 19 \quad \dots(ii)$$

Multiply equation (ii) by 4 on both sides, we find

$$4x + 20y = 76 \quad \dots(iii)$$

Subtracting equation (i) from equation (iii), we have

$$17y = 51$$

$$\Rightarrow y = \frac{51}{17} = 3$$

Substituting value of y in equation (i), we get

$$4x + 3 \times 3 = 25$$

$$4x = 16 \Rightarrow x = \frac{16}{4} = 4$$

∴ $x = 4$ and $y = 3$ is the solution.

Two Linear Equation in Two Unknowns : Now consider two linear equations in two unknowns,

$$a_1x + b_1y = c_1 \quad \dots(i)$$

$$a_2x + b_2y = c_2 \quad \dots(ii)$$

The above equations are nothing else but equations of 2 lines.

Any pair (x, y) which satisfy both the equation is called a solution to the above system of equations.

Solutions	Feature	Example
One	Lines intersect at $(1, -2)$ each other (i) $x + y = -1$ (ii) $x - y = 3$	
No solution	Lines are parallel to each other (i) $x + y = -1$ (ii) $2x + 2y = -6$	
Infinite (all points lying on the given line)	Both the lines coincide each other (i) $x + y = 1$ (ii) $2x + 2y = 2$	

Systems of Linear Equation :

Consistent System : A system (of 2 or 3 or more equations taken together) of linear equations is said to be consistent, if it has at least one solution.

Inconsistent System : A system of simultaneous linear equations is said to be inconsistent, if it has no solutions at all.

For example : $X + Y = 9$; $3X + 3Y = 8$

Clearly there are no values of X & Y which simultaneously satisfy the given equations. So the system is inconsistent.

REMEMBER

The system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has :

a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

No solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

The homogeneous system $a_1x + b_1y = 0$ and

$a_2x + b_2y = 0$ has the only solution $x = y = 0$ when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has a non-zero solution only when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, and in this case, the system has an infinite number of solutions.

Example 8 Find k for which the system $6x - 2y = 3$, $kx - y = 2$ has a unique solution.

Solution : The given system will have a unique solution,

$$\text{if } \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ i.e., } \frac{6}{k} \neq \frac{-2}{-1} \text{ or } k \neq 3.$$

Example 9 What is the value of k for which the system $x + 2y = 3$, $5x + ky = -7$ is inconsistent?

Solution : The given system will be inconsistent if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\text{i.e., if } \frac{1}{5} = \frac{2}{k} \neq \frac{3}{-7} \text{ or } k = 10.$$

Example 10 Find k such that the system $3x + 5y = 0$, $kx + 10y = 0$ has a non-zero solution.

Solution : The given system has a non zero solution,

$$\text{if } \frac{3}{k} = \frac{5}{10} \text{ or } k = 6$$

QUADRATIC EQUATION

An equation of the degree two of one variable is called quadratic equation.

General form : $ax^2 + bx + c = 0$... (i)

where a , b and c are all real number and $a \neq 0$.

For Example :

$$2x^2 - 5x + 3 = 0; \quad 2x^2 - 5 = 0; \quad x^2 + 3x = 0$$

A quadratic equation gives two and only two values of the unknown variable and both these values are called the roots of the equation. The roots of the quadratic equation (i) can be evaluated using the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \dots (ii)$$

The above formula provides both the roots of the quadratic equation, which are generally denoted by α and β ,

$$\text{say } \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

DISCRIMINANT

The expression inside the square root $b^2 - 4ac$ is called the **DISCRIMINANT** of the quadratic equation and denoted by D . Thus, Discriminant (D) = $b^2 - 4ac$.

Example 11 Which of the following is a quadratic equation?

- (a) $\frac{1}{x^2} + 2x + 3 = 0$
- (b) $(x-1)(x+4) = x^2 + 1$
- (c) $x^4 - 3x + 5 = 0$
- (d) $(2x+1)(3x-4) = 2x^2 + 3$

Solution :

- (d) Equations in options (a) and (c) are not quadratic equations as in (a) maximum power of x is fractional and in (c), it is not 2 in any of the terms.

For option (b), $(x-1)(x+4) = x^2 + 1$

$$\text{or } x^2 + 4x - x - 4 = x^2 + 1$$

$$\text{or } 3x - 5 = 0$$

which is not a quadratic equations but a linear.

For option (d), $(2x+1)(3x-4) = 2x^2 + 3$

$$\text{or } 6x^2 - 8x + 3x - 4 = 2x^2 + 3$$

$$\text{or } 4x^2 - 5x - 7 = 0$$

which is clearly a quadratic equation.

Solving a Quadratic Equation by Factorisation

STEP I : Express the equation in the standard form, i.e. $ax^2 + bx + c = 0$.

STEP II : Factorise the expression $ax^2 + bx + c$.

STEP III : Put each of the factors equal to zero and find the values of x .

These values of x are solutions or roots of the quadratic equation.

Example 12 Solve $x - \frac{1}{x} = 1\frac{1}{2}$

$$\text{Solution : } x - \frac{1}{x} = 1\frac{1}{2} \Rightarrow \frac{x^2 - 1}{x} = \frac{3}{2}$$

$$\Rightarrow 2(x^2 - 1) = 3x$$

$$\Rightarrow 2x^2 - 2 = 3x$$

$$\Rightarrow 2x^2 - 3x - 2 = 0$$

$$\Rightarrow 2x^2 - 4x + x - 2 = 0$$

$$\Rightarrow 2x(x-2)+1(x-2)=0$$

$$\Rightarrow (2x+1)(x-2)=0$$

$$\text{Either } 2x+1=0 \text{ or } x-2=0$$

$$\Rightarrow 2x=-1 \text{ or } x=2$$

$$\Rightarrow x = \frac{-1}{2} \text{ or } x=2$$

$$\therefore x = \frac{-1}{2}, 2 \text{ are solutions.}$$

Nature of Roots :

The nature of roots of the equation depends upon the nature of its discriminant D .

If $D < 0$, then the roots are non-real complex. Such roots are always conjugate to one another. That is, if one root is $p+iq$ then other is $p-iq$, $q \neq 0$.

If $D = 0$, then the roots are real and equal. Each root of the

equation becomes $-\frac{b}{2a}$. Equal roots are referred as repeated roots or double roots also.

If $D > 0$ then the roots are real and unequal.

In particular, if a, b, c are rational number, $D > 0$ and D is a perfect square, then the roots of the equation are rational number and unequal.

If a, b, c are rational number, $D > 0$ but D is not a perfect square, then the roots of the equation are irrational (surd). Surd roots are always conjugate to one another, that is if one root is $p+\sqrt{q}$, then the other is $p-\sqrt{q}$, $q > 0$.

If $a = 1$, b and c are integers, $D > 0$ and perfect square, then the roots of the equation are integers.

Sign of Roots :

Let α, β are real roots of the quadratic equation $ax^2+bx+c=0$ that is $D = b^2-4ac \geq 0$. Then

- Both the roots are positive if a and c have the same sign and the sign of b is opposite.
- Both the roots are negative if a, b and c all have the same sign.
- The roots have opposite sign if sign of a and c are opposite.
- The roots are equal in magnitude and opposite in sign if $b = 0$ [that is its roots α and $-\alpha$]
- The roots are reciprocal if $a = c$.

[that is the roots are α and $\frac{1}{\alpha}$]

- If $c = 0$, then one root is zero.
- If $b = c = 0$, then both the roots are zero.
- If $a = 0$, then one root is infinite.
- If $a = b = 0$, then both the roots are infinite.
- If $a = b = c = 0$, then the equation becomes an identity
- If $a + b + c = 0$ then one root is always unity and the other

root is $\frac{c}{a}$. Hence the roots are rational provided a, b, c are rational.

Example 13 Which of the following equations has real roots ?

$$(a) \quad 3x^2+4x+5=0 \quad (b) \quad x^2+x+4=0$$

$$(c) \quad (x-1)(2x-5)=0 \quad (d) \quad 2x^2-3x+4=0$$

Solution : (c) Roots of a quadratic equation

$$ax^2+bx+c=0 \text{ are real if } b^2-4ac \geq 0$$

Let us work with options as follows.

$$\text{Option (a) : } 3x^2+4x+5=0$$

$$b^2-4ac = (4)^2-4(3)(5) = -44 < 0.$$

Thus, roots are not real.

$$\text{Option (b) : } x^2+x+4=0$$

$$b^2-4ac = (1)^2-4(1)(4) = 1-16 = -15 < 0$$

Thus, roots are not real.

$$\text{Option (c) : } (x-1)(2x-5)=0 \Rightarrow 2x^2-7x+5=0$$

$$b^2-4ac = (-7)^2-4 \times 2 \times 5 = 49-40 = 9 > 0$$

Thus the roots are real.

$$\text{or } x=1 \text{ and } x=\frac{5}{2} > 0; \text{ Thus, equation has real roots.}$$

$$\text{Option (d) : } 2x^2-3x+4=0$$

$$b^2-4ac = (-3)^2-4(2)(4) = 9-32 = -23 < 0$$

Thus, roots are not real.

Hence, option (c) is correct.

Example 14 If $2x^2-7xy+3y^2=0$, then the value of $x:y$ is :

$$(a) \quad 3:2 \quad (b) \quad 2:3$$

$$(c) \quad 3:1 \text{ or } 1:2 \quad (d) \quad 5:6$$

Solution : (c) $2x^2-7xy+3y^2=0$

$$2\left(\frac{x}{y}\right)^2 - 7\left(\frac{x}{y}\right) + 3 = 0$$

$$\frac{x}{y} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{7 \pm \sqrt{49-24}}{2 \times 2} = \frac{7 \pm 5}{4} = 3, \frac{1}{2}$$

$$\Rightarrow \frac{x}{y} = \frac{3}{1} \text{ or } \frac{x}{y} = \frac{1}{2}$$

Symmetric Functions of Roots : An expression in α, β is called a symmetric function of α, β if the function is not affected by interchanging α and β . If α, β are the roots of the quadratic equation

$$ax^2+bx+c=0, a \neq 0 \text{ then,}$$

$$\text{Sum of roots : } \alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} \text{ and}$$

$$\text{Product of roots : } \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Above relations hold for any quadratic equation whether the coefficients are real or non-real complex.

With the help of above relations many other symmetric functions of α and β can be expressed in terms of the coefficients a , b and c .

Recurrence Relation

$$\alpha^n + \beta^n = (\alpha + \beta)(\alpha^{n-1} + \beta^{n-1}) - \alpha\beta(\alpha^{n-2} + \beta^{n-2})$$

Some symmetric functions of roots are

$$(i) \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$(ii) \alpha - \beta = \pm \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$(iii) \alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$(iv) \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$(v) \alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$$

$$(vi) \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 \\ = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$$

$$(vii) \alpha^4 - \beta^4 = (\alpha^2 + \beta^2)(\alpha^2 - \beta^2) \\ = [(\alpha + \beta)^2 - 2\alpha\beta](\alpha + \beta)(\alpha - \beta)$$

Formation of Quadratic Equation with Given Roots :

An equation whose roots are α and β can be written as $(x - \alpha)(x - \beta) = 0$

$$\text{or } x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\text{or } x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0.$$

Further if α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then

$$ax^2 + bx + c = a(x - \alpha)(x - \beta) \text{ is an identity.}$$

A number of relations between the roots can be derived using this identity by substituting suitable values of x real or imaginary.

Condition of a Common Root between Two Quadratic Equations :
Consider two quadratic equations

$$a_1x^2 + b_1x + c_1 = 0 \quad \dots(i)$$

$$\text{and } a_2x^2 + b_2x + c_2 = 0 \quad \dots(ii)$$

Let α be a common root of the two equations

$$\text{Then } a_1\alpha^2 + b_1\alpha + c_1 = 0 \text{ and } a_2\alpha^2 + b_2\alpha + c_2 = 0$$

On solving we get

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\text{or } \alpha^2 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \alpha = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Which gives the common root as well as the condition for common root.

Condition that Two Quadratic Equations have both the Roots Common :

Suppose that the equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have both the roots common.

$$\text{then } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

If the coefficients of two quadratic equations are rational (real) and they have one irrational (imaginary) root common then they must have both the roots common as such roots occur in conjugate pair.

Example 15 Form a quadratic equation, whose roots are 2 and -15 ?

$$\text{Solution : Sum of roots} = 2 - 15 = -13$$

$$\text{Product of roots} = 2 \times (-15) = -30$$

Required equation

$$= x^2 - x(\text{sum of roots}) + \text{product of roots} = 0$$

$$\Rightarrow x^2 + 13x - 30 = 0$$

Example 16 If a and b are the roots of the equation

$$x^2 - 6x + 6 = 0, \text{ then the find value of } a^2 + b^2.$$

$$\text{Solution : The sum of roots} = a + b = 6$$

$$\text{Product of roots} = ab = 6$$

$$\text{Now, } a^2 + b^2 = (a + b)^2 - 2ab = 36 - 12 = 24$$

Example 17 If a, b are the two roots of a quadratic equation such that $a + b = 24$ and $a - b = 8$, then the find quadratic equation having a and b as its roots.

$$\text{Solution : } a + b = 24 \text{ and } a - b = 8$$

$$\Rightarrow a = 16 \text{ and } b = 8 \Rightarrow ab = 16 \times 8 = 128$$

A quadratic equation with roots a and b is

$$x^2 - (a + b)x + ab = 0 \text{ or } x^2 - 24x + 128 = 0$$

INEQUATIONS : A statement or equation which states that one thing is not equal to another, is called an inequation.

Symbols :

'<' means "is less than"

'>' means "is greater than"

'≤' means "is less than or equal to"

'≥' means "is greater than or equal to"

For example :

$$(a) \ x < 3 \text{ means } x \text{ is less than } 3.$$

$$(b) \ y \geq 9 \text{ means } y \text{ is greater than or equal to } 9.$$

PROPERTIES

Adding the same number to each side of an equation does not effect the sign of inequality, it remains same, i.e.

if $x > y$ then, $x + a > y + a$.

Subtracting the same number to each side of an inequation does not effect the sign of inequality, i.e., if $x < y$ then, $x - a < y - a$.

Multiplying each side of an inequality with same number does not effect the sign of inequality, i.e., if $x \leq y$ then $ax \leq ay$ (where, $a > 0$).

Multiplying each side of an inequality with a negative number affects the sign of inequality or sign of inequality reverses, i.e., if $x < y$ then $ax > ay$ (where $a < 0$).

Dividing each side of an inequation by a positive number does not affect the sign of inequality, i.e., if $x \leq y$ then

$$\frac{x}{a} \leq \frac{y}{a} \text{ (where } a > 0 \text{)}.$$

Dividing each side of an inequation by a negative number reverses the sign of inequality, i.e., if $x > y$ then

$$\frac{x}{a} < \frac{y}{a} \text{ (where } a < 0 \text{)}.$$

REMEMBER

If $a > b$ and a, b, n are positive, then $a^n > b^n$ but $a^{-n} < b^{-n}$.
For example $5 > 4$; then $5^3 > 4^3$ or $125 > 64$, but

$$5^{-3} < 4^{-3} \text{ or } \frac{1}{125} < \frac{1}{64}.$$

If $a > b$ and $c > d$, then $(a + c) > (b + d)$.

If $a > b > 0$ and $c > d > 0$, then $ac > bd$.

If the signs of all the terms of an inequality are changed, then the sign of the inequality will also be reversed.

MODULUS:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

- If a is positive real number, x and y be the fixed real numbers, then
 - $|x - y| < a \Leftrightarrow y - a < x < y + a$
 - $|x - y| \leq a \Leftrightarrow y - a \leq x \leq y + a$
 - $|x - y| > a \Leftrightarrow x > y + a \text{ or } x < y - a$
 - $|x - y| \geq a \Leftrightarrow x \geq y + a \text{ or } x \leq y - a$
- Triangle inequality:
 - $|x + y| \leq |x| + |y|, \forall x, y \in R$
 - $|x - y| \geq |x| - |y|, \forall x, y \in R$

Example 18 If $a - 8 = b$, then determine the value of $|a - b| - |b - a|$.

Solution: $|a - b| = |8| = 8$

$$\Rightarrow |b - a| = |-8| = 8 \Rightarrow |a - b| - |b - a| = 8 - 8 = 0$$

Example Solve: $3x + 4 \leq 19, x \in N$

Solution: $3x + 4 \leq 19$

$$3x + 4 - 4 \leq 19 - 4 \text{ [Subtracting 4 from both the sides]}$$

$$3x \leq 15$$

$$\frac{3x}{3} \leq \frac{15}{3} \quad \text{[Dividing both the sides by 3]}$$

$$x \leq 5; x \in N$$

$$\therefore x = \{1, 2, 3, 4, 5\}.$$

APPLICATIONS

Formulation of Equations / Expressions :

A formula is an equation, which represents the relations between two or more quantities.

For example :

Area of parallelogram (A) is equal to the product of its base (b) and height (h), which is given by

$$A = b \times h$$

$$\text{or } A = bh.$$

Perimeter of triangle (P),

$$P = a + b + c, \text{ where } a, b \text{ and } c \text{ are three sides.}$$

Example 20 Form the expression for each of the following :

- 5 less than a number is 7.
- Monika's salary is 1500 less than thrice the salary of Surbhi.

Solution : (a) Expression is given by

$$x - 5 = 7, \text{ where } x \text{ is any number}$$

- Let the salary of Surbhi be 'x' and salary of Monika be 'y'.

Now, according to the question

$$y = 3x - 1500$$

AVERAGE

'Average' is a very simple but effective way of representing an entire group by a single value.

$$\text{Average or Mean} = \frac{\text{Sum of given quantities}}{\text{Number of quantities}}$$

To calculate the sum of quantities, they should be in the same unit.

Example 21 The average of the first nine prime numbers is:

- 9
- 11
- $11\frac{1}{9}$
- $11\frac{2}{9}$

Solution :

- Average

$$= \left(\frac{2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23}{9} \right) = \frac{100}{9} = 11\frac{1}{9}.$$

Example 22 In three numbers, the first is twice the second and thrice the third. If the average of these three numbers is 44, then the first number is :

Solution :

- (a) Let the three numbers be x , y and z

Therefore, $x = 2y = 3z$,

$$y = \frac{x}{2} \text{ and } z = \frac{x}{3}$$

$$\text{Now, } \frac{x + \frac{x}{2} + \frac{x}{3}}{3} = 44$$

$$\text{or } \frac{11x}{18} = 44 \text{ or } x = 72$$

Example 23 The average of five consecutive odd numbers is 61.

What is the difference between the highest and lowest numbers?

- (a) 2 (b) 5
(c) 8 (d) Cannot be determined

Solution :

- (c) Let the numbers be x , $x+2$, $x+4$, $x+6$ and $x+8$.

$$\text{Then, } \frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 61$$

$$\text{or } 5x + 20 = 305 \text{ or } x = 57.$$

$$\text{So, required difference} = (57 + 8) - 57 = 8.$$

Average of a group consisting two different groups when their averages are known :

Let Group A contains m quantities and their average is a and Group B contains n quantities and their average is b , then average of group C containing $a + b$ quantities

$$= \frac{ma + nb}{m + n}$$

Example 24 There are 30 student in a class. The average age of the first 10 students is 12.5 years. The average age of the next 20 students is 13.1 years. The average age of the whole class is :

- (a) 12.5 years (b) 12.7 years
(c) 12.8 years (d) 12.9 years

Solution :

- (d) Total age of 10 students = $12.5 \times 10 = 125$ years
Total age of 20 students = $13.1 \times 20 = 262$ years

$$\text{Average age of 30 students} = \frac{125 + 262}{30} = 12.9 \text{ years}$$

Example 25 The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is

- (a) 1 : 2 (b) 2 : 3
(c) 3 : 4 (d) 3 : 5

Solution :

- (b) Let the number of boys in a class be x .
Let the number of girls in a class be y .
 $\therefore$ Sum of the ages of the boys = $16.4x$

Sum of the ages of the girls = $15.4y$

$$\therefore 15.8(x+y) = 16.4x + 15.4y$$

$$\Rightarrow 0.6x = 0.4y \Rightarrow \frac{x}{y} = \frac{2}{3}$$

$$\therefore \text{Required ratio} = 2 : 3$$

Example 26 The average monthly expenditure of a family was Rs. 2200 during the first 3 months; Rs. 2250 during the next 4 months and Rs. 3120 during the last 5 months of a year. If the total saving during the year were Rs. 1260, then the average monthly income was

- (a) Rs. 2605 (b) Rs. 2805
(c) Rs. 2705 (d) Rs. 2905

Solution :

- (c) Total annual income
= $3 \times 2200 + 4 \times 2250 + 5 \times 3120 + 1260$
= $6600 + 9000 + 15600 + 1260 = 32460$

$$\therefore \text{Average monthly income} = \frac{32460}{12} = \text{Rs. } 2705$$

If, in a group, one or more new quantities are added or excluded, then the new quantity or sum of added or excluded quantities = [Change in no. of quantities $\times$ original average] $\pm$ [change in average $\times$ final no. of quantities]
Take +ve sign if quantities added and
take -ve sign if quantities removed.

Example 27 The average weight of 29 students in a class is 48 kg. If the weight of the teacher is included, the average weight rises by 500 g. Find the weight of the teacher.

Solution :

Here, weight of the teacher is added and final average of the group increases.

$\therefore$ Change in average is (+)ve, using the formula

Sum of the quantities added

$$= \left(\frac{\text{Change in no. of quantities}}{\times} \right) + \left(\frac{\text{Change in average}}{\times} \right)$$

$$\Rightarrow \text{weight of teacher} = (1 \times 48) + (0.5 \times 30) = 63 \text{ kg.}$$

$$\therefore \text{weight of teacher is 63 kg.}$$

Example 28 The average age of 40 students in a class is 15 years. When 10 new students are admitted, the average is increased by 0.2 year. Find the average age of the new students.

Solution :

Here, 10 new students are admitted.

$\therefore$ change in average is +ve. Using the formula

Sum of the quantities added

$$= \left(\frac{\text{Change in no. of quantities}}{\times} \right) + \left(\frac{\text{Change in average}}{\times} \right)$$

Chapter 08: Percentage

Increase Percent

The word "per cent" is derived from the latin words "per centum", which means "per hundred".

A percentage is a fraction with denominator hundred.

It is denoted by the symbol %.

Numerator of the fraction is called the rate per cent.

Let Mohan scores 125 in CSAT Paper-I and 115 in CSAT - Paper II out of 200 marks in each. Then as a total in both he scored $125 + 115 = 240$ and total full marks $= 200 + 200 = 400$.

∴ out of 400 marks he scored 240

∴ out of 1 marks he scored $\frac{240}{400}$

∴ out of 100 marks he scored $\frac{240}{400} \times 100 = 60$

Hence we say he scored 240 out of 400 or he scored 60% of 400

Value of Percentage :

Value of percentage always depends on the quantity to which it refers.

Consider the statement :

"65% of the students in this class are boys". From the context, it is understood that boys form 65% of the total number of students in the class. To know the value of 65%, the value of the total number of student should be known.

If the total number of students is 200, then,

the number of boys $= \frac{200 \times 65}{100} = 130$;

It can also be written as

$$(200) \times (0.65) = 130.$$

To Express the Fraction Equivalent to % :

Express the fraction with the denominator 100, then the numerator is the answer.

Let we have to convert the fraction $\frac{3}{4}$ into its equivalent

$$\text{percentage } \frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\%$$

$$\text{Also } \frac{3}{5} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60\%$$

we also convert $\frac{3}{4}$ directly into percentage by multiplying $\frac{3}{4}$ with

$$100. \quad \frac{3}{4} \text{ in percent} = \frac{3}{4} \times 100 = 75\%$$

Example 1 Express the fraction $\frac{11}{12}$ into the per cent.

$$\text{Solution : } \frac{11}{12} = \frac{11 \times 100}{12 \times 100} = \frac{91\frac{2}{3}}{100} = 91\frac{2}{3}\%$$

To express % equivalent to fraction :

$$a\% = \frac{a}{100}$$

Example 2 Express $45\frac{5}{6}\%$ into fraction.

$$\text{Solution : } 45\frac{5}{6}\% = \frac{45\frac{5}{6}}{100} = \frac{275}{6 \times 100} = \frac{11}{24}$$

$$\text{Increase \%} = \frac{\text{Increase value}}{\text{Original value}} \times 100$$

Fractional Equivalents of %

$$1\% = \frac{1}{100}$$

$$2\% = \frac{1}{50}$$

$$4\% = \frac{1}{25}$$

$$5\% = \frac{1}{20}$$

$$6\frac{1}{4}\% = \frac{1}{16}$$

$$10\% = \frac{1}{10}$$

$$11\frac{1}{3}\% = \frac{17}{150}$$

$$33\frac{1}{3}\% = \frac{1}{3}$$

$$40\% = \frac{2}{5}$$

$$50\% = \frac{1}{2}$$

$$66\frac{2}{3}\% = \frac{2}{3}$$

$$60\% = \frac{3}{5}$$

$$75\% = \frac{3}{4}$$

$$80\% = \frac{4}{5}$$

$$12\frac{1}{2}\% = \frac{1}{8}$$

$$96\% = \frac{24}{25}$$

$$16\% = \frac{4}{25}$$

$$100\% = 1$$

$$16\frac{2}{3}\% = \frac{1}{6}$$

$$115\% = \frac{23}{20}$$

$$20\% = \frac{1}{5}$$

$$133\frac{1}{3}\% = \frac{4}{3}$$

$$25\% = \frac{1}{4}$$

Example 3 Rent of the house is increased from ₹ 7000 to ₹ 7700. Express the increase in price as a percentage of the original rent.

Solution : Increase value = ₹ 7700 – ₹ 7000 = ₹ 700

$$\text{Increase \%} = \frac{\text{Increase value}}{\text{Original value}} \times 100 = \frac{700}{7000} \times 100 = 10$$

∴ Percentage rise = 10 %.

Decrease Percent

$$\text{Decrease \%} = \frac{\text{Decrease value}}{\text{Original value}} \times 100$$

Example 4 The cost of a bike last year was ₹ 19000. Its cost this year is ₹ 17000. Find the per cent decrease in its cost.

$$\begin{aligned} \text{Solution : \% decrease} &= \frac{19000 - 17000}{19000} \times 100 \\ &= \frac{2000}{19000} \times 100 = 10.5\% \end{aligned}$$

∴ Per cent decrease = 10.5 %.

If A is x % of C and B is y % of C, then A is $\frac{x}{y} \times 100$ % of B.

Two numbers are respectively 20% and 25% of third number. What percentage is the first of the second ?

$$\text{The required percentage} = \frac{20}{25} \times 100 = 80\%$$

Example 5 A positive number is divided by 5 instead of being multiplied by 5. By what per cent is the result of the required correct value ?

Solution : Let the number be 1, then the correct answer = 5

$$\text{The incorrect answer that was obtained} = \frac{1}{5}$$

$$\therefore \text{The required \%} = \frac{1}{5 \times 5} \times 100 = 4\%$$

(a) If two numbers are respectively x% and y% more than a third number, then the first number is $\left(\frac{100+x}{100+y} \times 100\right)$ % of the second and the second is $\left(\frac{100+y}{100+x} \times 100\right)$ % of the first.

(b) If two number are respectively x% and y% less than a third number, then the first number is $\left(\frac{100-x}{100-y} \times 100\right)$ % of the second and the second is $\left(\frac{100-y}{100-x} \times 100\right)$ % of the first.

Example 6 Two numbers are respectively 30% and 60% more than a third number. What percent is the first of the second?

Solution : Given x = 30, y = 60% $\left(\frac{100+x}{100+y} \times 100\right)$ % of the second

$$\begin{aligned} \therefore \text{First number} &= \left(\frac{100+30}{100+60} \times 100\right) \% \text{ of the second} \\ &= 81.25\% \text{ of the second.} \end{aligned}$$

Example 7 Two number are respectively 32% and 20% less than a third number. What percent is the first of the second ?

Solution : Given, x = 32, y = 20

$$\begin{aligned} \therefore \text{First number} &= \left(\frac{100-x}{100-y} \times 100\right) \% \text{ of the second} \\ &= \left(\frac{100-32}{100-20} \times 100\right) \% \text{ of the second} \\ &= 85\% \text{ of the second.} \end{aligned}$$

x % of a quantity is taken by the first, y % of the remaining is taken by the second and z % of the remaining is taken by third person. Now, if A is left in the fund, then the initial amount

$$= \frac{A \times 100 \times 100 \times 100}{(100-x)(100-y)(100-z)} \text{ in the beginning.}$$

x % of a quantity is added. Again, y % of the increased quantity is added. Again z % of the increased quantity is added. Now it becomes A, then the initial amount

$$= \frac{A \times 100 \times 100 \times 100}{(100+x)(100+y)(100+z)}$$

Example 8 3.5 % income is taken as tax and 12.5 % of the remaining is saved. This leaves ₹ 4,053 to spend. What is the income ?

Solution : By direct method,

$$\text{Income} = \frac{4053 \times 100 \times 100}{(100-3.5)(100-12.5)} = ₹ 4800.$$

If the price of a commodity increases by $r\%$, then reduction in consumption, so as not to increase the expenditure is

$$\left(\frac{r}{100+r} \times 100\right)\%$$

If the price of a commodity decreases by $r\%$, then the increase in consumption so as not to decrease the expenditure is

$$\left(\frac{r}{100-r} \times 100\right)\%$$

Example 9 If the price of coal be raised by 20%, then find by how much a householder must reduce his consumption of this commodity so as not to increase his expenditure?

Solution : Reduction in consumption

$$\begin{aligned} &= \left(\frac{20}{100+20} \times 100\right)\% \\ &= \left(\frac{20}{120} \times 100\right)\% = 16.67\% \end{aligned}$$

Population Formula

If the original population of a town is P , and the annual increase is $r\%$, then the population after n years is

$$P\left(1 + \frac{r}{100}\right)^n$$

and population before n years

$$= \frac{P}{\left(1 + \frac{r}{100}\right)^n}$$

If the annual decrease be $r\%$, then the population after n years is

$$P\left(1 - \frac{r}{100}\right)^n$$

and population before n years

$$= \frac{P}{\left(1 - \frac{r}{100}\right)^n}$$

Example 10 The population of a certain town increased at a certain rate per cent per annum. Now it is 456976. Four years ago, it was 390625. What will it be 2 years later?

Solution : Suppose the population increases at $r\%$ per annum.

$$\text{Then, } 390625\left(1 + \frac{r}{100}\right)^4 = 456976$$

$$\therefore \left(1 + \frac{r}{100}\right)^2 = \sqrt{\frac{456976}{390625}} = \frac{676}{625}$$

Population 2 years later

$$= 456976\left(1 + \frac{r}{100}\right)^2$$

$$= 456976 \times \frac{676}{625} = 494265 \text{ (approx)}$$

Example 11 The population of a city increases at the rate of 4% per annum. There is an additional annual increase of 1% in the population due to the influx of job seekers. Therefore, find the % increase in the population after 2 years.

Solution : The net annual increase = 5%.

Let the initial population be 100.

Then, population after 2 years

$$= 100 \times 1.05 \times 1.05$$

$$= 110.25$$

Therefore, % increase in population

$$= (110.25 - 100) = 10.25\%$$

If a number A is increased successively by $x\%$ followed by $y\%$ and then $z\%$, then the final value of A will be

$$A\left(1 + \frac{x}{100}\right)\left(1 + \frac{y}{100}\right)\left(1 + \frac{z}{100}\right)$$

In case a given value decreases by an percentage then we will use negative sign before that.

Example 12 The population of a town is 1,44,000. It increases by 5% during the first year. During the second year, it decreases by 10% and increases by 15% during the third year. What is the population after 3 years.

Solution : Given, $P = 1,44,000$

$$x = 5\%$$

$$y = 10\%$$

$$z = 15\%$$

$\therefore$ Population after 3 years

$$= 1,44,000 \left(1 + \frac{5}{100}\right)\left(1 - \frac{10}{100}\right)\left(1 + \frac{15}{100}\right)$$

$$= \frac{1,44,000 \times 105 \times 90 \times 115}{100 \times 100 \times 100} = 156492$$

First Increase and Then Decrease :

If the value is first increased by $x\%$ and then decreased by

$y\%$ then there is $\left(x - y - \frac{xy}{100}\right)\%$ increase or decrease,

according to the +ve or -ve sign respectively.

If the value of a number is increased by $x\%$ and then decreased by $x\%$, then there is only net decrease which is

$$\text{equal to } \left(\frac{x}{10}\right)^2$$

Example 13 A number is increased by 10% and then it is decreased by 10%. Find the net increase or decrease percent.

$$\text{Solution : \% change} = 10 - 10 - \frac{10 \times 10}{100} = -1\%$$

i.e., 1% decrease.

Chapter 09: Profit and Loss

Cost Price : The amount paid to purchase an article or the price at which an article is made, is known as its cost price.

The cost price is abbreviated as C.P.

Selling Price : The price at which an article is sold, is known as its selling price.

The selling price is abbreviated as S.P.

Profit : If the selling price (S.P.) of an article is greater than the cost price (C.P.), then the difference between the selling price and cost price is called profit.

Thus, If $S.P. > C.P.$, then

$$\text{Profit} = S.P. - C.P.$$

$$\Rightarrow S.P. = C.P. + \text{Profit}$$

$$\Rightarrow C.P. = S.P. - \text{Profit}$$

Loss : If the selling price (S.P.) of an article is less than the cost price (C.P.), then the difference between the cost price (C.P.) and the selling price (S.P.) is called loss.

Thus, if $S.P. < C.P.$, then

$$\text{Loss} = C.P. - S.P.$$

$$\Rightarrow C.P. = S.P. + \text{Loss}$$

$$\Rightarrow S.P. = C.P. - \text{Loss}$$

Example 1 An article was bought for ₹ 2000 and sold for ₹ 2200. Find the gain or loss.

Solution : C.P. of the article = ₹ 2000

S.P. of the article = ₹ 2200

Since $S.P. > C.P.$ So there is gain.

Gain (profit) = $S.P. - C.P.$

$$= ₹ 2200 - ₹ 2000 = ₹ 200$$

Profit and Loss Percentage

The profit per cent is the profit that would be obtained for a C.P. of ₹ 100.

$$\text{Profit per cent} = \frac{\text{Profit}}{C.P.} \times 100$$

Similarly, the loss per cent is the loss that would be made for a C.P. of ₹ 100.

$$\text{Loss per cent} = \frac{\text{Loss}}{C.P.} \times 100$$

REMEMBER

$$\text{Profit} = \frac{C.P. \times \text{Profit \%}}{100}$$

$$\text{Loss} = \frac{C.P. \times \text{Loss \%}}{100}$$

$$S.P. = \left(\frac{100 + \text{Profit \%}}{100} \right) \times C.P.$$

$$S.P. = \left(\frac{100 - \text{Loss \%}}{100} \right) \times C.P.$$

$$C.P. = \frac{100 \times S.P.}{100 + \text{Profit \%}}$$

$$C.P. = \frac{100 \times S.P.}{100 - \text{Loss \%}}$$

Example 2 A cycle was purchased for ₹ 1600 and sold for ₹ 1400. Find the loss and loss %.

Solution : C.P. of the cycle = ₹ 1600

S.P. of the cycle = ₹ 1400

Since $S.P. < C.P.$, so there is loss.

Loss = $C.P. - S.P.$

$$= ₹ 1600 - ₹ 1400 = ₹ 200.$$

$$\text{Loss \%} = \frac{\text{Loss}}{C.P.} \times 100 = \frac{200}{1600} \times 100 = 12\frac{1}{2}\%$$

Example 3 By selling a table for ₹ 330, a trader gains 10%. Find the cost price of the table.

Solution : S.P. = ₹ 330, Gain = 10%

$$\begin{aligned} \therefore C.P. &= \left(\frac{100}{100 + \text{Gain \%}} \right) \times S.P. \\ &= \frac{100}{100 + 10} \times 330 \\ &= \frac{100}{110} \times 330 = ₹ 300. \end{aligned}$$

Example 4 A sells a bicycle to B at a profit of 20% and B sells it to C at a profit of 25%. If C pays ₹ 225 for it, what did A pay for it.

$$\text{Solution : } C.P. \text{ of } A = 225 \times \frac{100}{100 + 20} \times \frac{100}{100 + 25}$$

$$= 225 \times \frac{100}{120} \times \frac{100}{125} = ₹ 150.$$

Example 5 A mobile phone is sold for ₹ 5060 at a gain of 10%. What would have been the gain or loss per cent if it had been sold for ₹ 4370 ?

Solution : S.P. = ₹ 5060, gain = 10%

$$\therefore C.P. = \frac{5060 \times 100}{100 + 10} = ₹ 4600.$$

2nd S.P. = ₹ 4370

Since, $S.P. < C.P.$, so there is loss.

$$\therefore \text{Loss \%} = \frac{(4600 - 4370) \times 100}{4600} = 5\%$$

If a man buys x items for ₹ y and sells z items for ₹ w , then the gain

or loss percent made by him is $\left(\frac{xw}{zy} - 1 \right) \times 100\%$

Example 6 Some articles were bought at 6 for ₹5 and sold at 5 for ₹6. Gain percent is –

Solution : Quantity Price
 6 5
 5 6

$$\% \text{ profit} = \left(\frac{xw}{zy} - 1 \right) \% = \left(\frac{6 \times 6}{5 \times 5} - 1 \right) \% = \frac{11}{25} \times 100 = 44\%$$

Dishonest Dealing

$$\text{Gain \%} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

$$\frac{\text{True Scale}}{\text{False Scale}} = \frac{100 + \text{gain\%}}{100 - \text{loss\%}}$$

A merchant uses $y\%$ less weight/length and sells his good at gain/loss of $x\%$. The overall % gain/loss is given by =

$$\left[\frac{y+x}{100-y} \times 100 \right] \%$$

Example 7 A shopkeeper sells the goods at 44% less on the cost price but uses 30% less weight. What is his percentage profit or loss?

Solution : Given, $x = -44$, $y = 30$

$$\text{Then, overall \% gain/loss} = \left(\frac{30-44}{100-30} \times 100 \right) \% = -20\%$$

His % loss is 20%.

Example 8 A cloth merchant says that due to slump in the market, he sells the cloth at 10% loss, but he uses a false metre-scale and actually gain 15%. Find the actual length of the scale.

Solution :

$$\frac{\text{True scale}}{\text{False scale}} = \frac{100 + \text{gain\%}}{100 - \text{loss\%}}$$

$$\frac{100}{\text{False scale}} = \frac{100+15}{100-10}$$

$$\Rightarrow \text{False scale} = \frac{100 \times 90}{115} = 78.26 \text{ cm}$$

Example 9 A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 960 g for the kg weight. Find his gain per cent.

Solution : Error = 1 kg – 960 g
 = 1000 g – 960 g = 40 g.

$$\begin{aligned} \therefore \text{Gain \%} &= \frac{40}{1000-40} \times 100 \\ &= \frac{40}{960} \times 100 = 4\frac{1}{6} \% \end{aligned}$$

Goods Passing through Successive Hands

When there are two successive profits of $a\%$ and $b\%$, then the resultant profit per cent is given by

$$\left(a + b + \frac{ab}{100} \right) \%$$

When there is a profit of $a\%$ and loss by $b\%$ in a transaction, then the resultant profit or loss per cent is given by

$$\left(a - b - \frac{ab}{100} \right) \%,$$

according to the +ve or -ve sign respectively.

When cost price and selling price are reduced by the same amount (A) and profit increases then cost price (C.P.)

$$= \frac{[\text{Initial profit \%} + \text{Increase in profit \%}] \times A}{\text{Increase in profit \%}}$$

Example 10 A table is sold at a profit of 20%. If the cost price and selling price are ₹ 200 less, the profit would be 8% more. Find the cost price.

Solution : By direct method,

$$\text{C.P.} = \frac{\text{₹}(20+8) \times 200}{8} = 28 \times 25 = \text{₹} 700.$$

If cost price of x articles is equal to the selling price of

$$y \text{ articles, then profit/loss percentage} = \frac{x-y}{y} \times 100 \%$$

according to +ve or -ve sign respectively.

Example 11 If the C.P. of 15 tables be equal to the S.P. of 20 tables, find the loss per cent.

Solution : By direct method,

$$\text{Profit/Loss \%} = \frac{-5}{20} \times 100$$

= 25% loss, since it is -ve.

Example 12 If the C.P. of 6 articles is equal to the S.P. of 4 articles. Find the gain per cent.

Solution : Let C.P. of an article be ₹ x ; then,

$$\text{C.P. of 6 articles} = \text{₹} 6x$$

$$\text{C.P. of 4 articles} = \text{₹} 4x$$

$$\text{But S.P. of 4 articles} = \text{C.P. of 6 articles}$$

$$\therefore \text{S.P. of 4 articles} = 6x$$

$$\text{Thus, gain} = \text{S.P.} - \text{C.P.} = (6x - 4x) = \text{₹} 2x$$

$$\therefore \text{Gain \%} = \frac{2x}{4x} \times 100 = 50$$

Chapter 10: Interest

Interest is the fixed amount paid on borrowed money.

The sum lent is called the **Principal**.

The sum of the principal and interest is called the **Amount**.

Interest is of two kinds :

Simple interest and Compound interest

SIMPLE INTEREST :

When interest is calculated on the original principal for any length of time, it is called simple interest.

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$$

$$\text{Amount} = \text{Principal} + \text{Simple Interest}$$

$$\text{Principal (P)} = \frac{100 \times \text{S.I.}}{R \times T}$$

$$\text{Rate (R)} = \frac{100 \times \text{S.I.}}{T \times P}$$

$$\text{Time (T)} = \frac{100 \times \text{S.I.}}{P \times R}$$

If rate of simple interest differs from year to year, then

$$\text{S.I.} = P \times \frac{(R_1 + R_2 + R_3 + \dots)}{100}$$

Example 1 Find the interest to be paid on a loan of ₹ 6000 at 5% per year for 5 years.

Solution : $P = ₹ 6000$, $R = 5\%$ and $T = 5$ years

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{6000 \times 5 \times 5}{100} = ₹ 1500$$

Example 2 Find the amount to be paid back on a loan of ₹ 18,000 at 5.5% per annum for 3 years.

Solution : $P = ₹ 18000$, $R = 5.5\%$, $T = 3$ years

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{18000 \times 5.5 \times 3}{100} = ₹ 2970$$

$$\text{Amount} = P + I = 18000 + 2970 = ₹ 20970$$

Example 3 In how many years will a sum of money triple itself, at 25% per annum simple interest.

Solution : Let the sum of money be ₹ P. So, $A = 3P$

$$\text{and S.I.} = A - P = 3P - P = 2P; R = 25\%$$

$$\therefore T = \frac{100 \times \text{S.I.}}{P \times R} = \frac{100 \times 2P}{P \times 25} = 8 \text{ years}$$

Example 4 What rate per cent per annum will produce ₹ 250 as simple interest on ₹ 6000 in 2.5 years ?

Solution : $P = ₹ 6000$; Time (T) = 2.5 years; S.I. = ₹ 250

$$\therefore \text{Rate} = \frac{\text{S.I.} \times 100}{P \times T} = \frac{250 \times 100}{6000 \times 2.5} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}\%$$

COMPOUND INTEREST :

Money is said to be lent at compound interest when at the end of a year or other fixed period, the interest that has become due is not paid to the lender, but is added to the sum lent, and the amount thus obtained becomes the principal in the next year or period. The process is repeated until the amount for the last period has been found. Hence, when the interest charged after a certain specified time period is added to form new principal for the next time period, the interest is said to be compounded and the total interest accrued is compound interest.

$$\text{C.I.} = P \left[\left(1 + \frac{r}{100} \right)^n - 1 \right];$$

$$\text{Amount (A)} = P \left(1 + \frac{r}{100} \right)^n$$

If rate of compound interest differs from year to year,

$$\text{then Amount} = P \left(1 + \frac{r_1}{100} \right) \left(1 + \frac{r_2}{100} \right) \left(1 + \frac{r_3}{100} \right) \dots$$

Example 5 Find the compound interest on ₹ 70000 for 4 years at the rate of 14% per annum compounded annually.

Solution : $P = ₹ 70000$, $n = 4$, $r = 14\%$

$$A = P \left(1 + \frac{r}{100} \right)^n = 70000 \left(1 + \frac{14}{100} \right)^4 = ₹ 118227.20$$

$$\text{C.I.} = A - P = 118227.20 - 70000 = ₹ 48227.20$$

Example 6 If ₹ 60000 amounts to ₹ 68694 in 2 years then find the rate of compound interest.

Solution : Given : $A = ₹ 68694$, $P = ₹ 60000$, $n = 2$ years, $r = ?$

$$\therefore A = P \left(1 + \frac{r}{100} \right)^n \quad \therefore 68694 = 60000 \left(1 + \frac{r}{100} \right)^2$$

$$\Rightarrow \frac{68694}{60000} = \left(1 + \frac{r}{100} \right)^2 \Rightarrow \frac{11449}{10000} = \left(1 + \frac{r}{100} \right)^2$$

$$\Rightarrow 1 + \frac{r}{100} = \sqrt{\frac{11449}{10000}} = \sqrt{1.1449}$$

$$\Rightarrow 1 + \frac{r}{100} = 1.07$$

$$\Rightarrow \frac{r}{100} = 1.07 - 1 = 0.07$$

$$\therefore r = 0.07 \times 100 = 7\%$$

Example 7 In how many years, the sum of ₹ 10000 will become ₹ 10920.25 if the rate of compound interest is 4.5% per annum?

Solution : $A = ₹ 10920.25$, $P = ₹ 10000$,

Rate of interest = 4.5%, Time (n) = ?

$$\therefore A = P \left(1 + \frac{r}{100} \right)^n$$

$$\therefore 10920.25 = 10000 \left(1 + \frac{4.5}{100} \right)^n$$

$$\frac{10920.25}{10000} = \left(1 + \frac{0.9}{20} \right)^n = \left(\frac{20.9}{20} \right)^n$$

$$\Rightarrow \frac{436.81}{400} = \left(\frac{20.9}{20} \right)^n$$

$$\Rightarrow \left(\frac{20.9}{20} \right)^2 = \left(\frac{20.9}{20} \right)^n$$

Hence ₹ 10000 will become ₹ 10920.25 in 2 years at 4.5%.

Example 8 Suppose ₹ 1000 is deposited for 6 years in an account paying 8.31% per year compounded annually.

(a) Find the compound amount.

(b) Find the amount of interest earned.

Solution : (a) In the formula above, $P = 1000$, $i = 0.0831$, and $n = 6$.

The compound amount is

$$A = P \left(1 + \frac{r}{100} \right)^n$$

$$A = 1000(1.0831)^6$$

$$A = ₹ 1614.40$$

(b) Subtract the initial deposit from the compound amount.

$$\text{Amount of interest} = ₹ 1614.40 - ₹ 1000 = ₹ 614.40$$

Compound Interest : when interest is compounded annually but time is in fraction

If time = $t \frac{p}{q}$ years, then

$$A = P \left(1 + \frac{r}{100} \right)^t \left(1 + \frac{\frac{p}{q}r}{100} \right)$$

Example 9 Find the compound interest on ₹ 8000 at 15% per annum for 2 years 4 months, compounded annually.

Solution : Time = 2 years 4 months = $2\frac{4}{12}$ years = $2\frac{1}{3}$ years

$$\text{Amount} = ₹ \left[8000 \left\{ \left(1 + \frac{15}{100} \right) \right\}^2 \left(1 + \frac{\frac{1}{3} \times 15}{100} \right) \right]$$

$$= ₹ \left(8000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20} \right) = ₹ 11109$$

$$\therefore \text{C.I.} = ₹ (11109 - 8000) = ₹ 3109$$

Compound Interest : when interest is calculated half-yearly
Since r is calculated half-yearly therefore the rate per cent will become half and the time period will become twice, i.e.,

Rate per cent when interest is paid half-yearly = $\frac{r}{2}\%$ and

time = $2 \times$ time given in years

Hence,

$$A = P \left(1 + \frac{r}{2 \times 100} \right)^{2n}$$

Example 10 What will be the compound interest on ₹ 4000 in 4 years at 8 per cent annum. If the interest is calculated half-yearly.

Solution : Given : $P = ₹ 4000$, $r = 8\%$, $n = 4$ years

Since interest is calculated half-yearly, therefore,

$$\therefore A = 4000 \left(1 + \frac{4}{2 \times 100} \right)^{2 \times 4}$$

$$= 4000 \times \left(\frac{26}{25} \right)^8$$

$$= 4000 \times 1.3685 = 5474.2762$$

$$\text{Amount} = ₹ 5474.28$$

$$\therefore \text{Interest} = \text{Amount} - \text{Principal}$$

$$= ₹ 5474.28 - ₹ 4000 = ₹ 1474.28$$

Compound Interest : when interest is calculated quarterly

Since 1 year has 4 quarters, therefore rate of interest will

become $\frac{1}{4}$ th of the rate of interest per annum, and the time

period will be 4 times the time given in years

Hence, for quarterly interest

$$A = P \left(1 + \frac{r}{4 \times 100} \right)^{4n}$$

when interest is compounded monthly

$$A = P \left[1 + \frac{r/12}{100} \right]^{12t} = P \left[1 + \frac{r}{1200} \right]^{12t}$$

Example 11 Find the compound interest on ₹ 25625 for 12 months at 16% per annum, compounded quarterly.

Solution : Principal (P) = ₹ 25625

Rate (r) = 16%

Time = 12 months = 1 year

$$A = 25625 \left(1 + \frac{16}{4 \times 100} \right)^{4 \times 1} = 25625 \left(\frac{26}{25} \right)^4 = ₹ 29977.62$$

$$\text{C.I.} = A - P = 29977.62 - 25625 = ₹ 4352.62$$

Chapter 11: Ratio and Proportion, Partnership

RATIO

Ratio is strictly a mathematical term to compare two similar quantities expressed in the same units.

The ratio of two terms 'x' and 'y' is denoted by $x : y$. In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y .

The numerator of the ratio is called the antecedent (x) and the denominator is called the consequent (y) of the ratio.

Comparison of Two or More Ratios : Two or more ratios may be compared by reducing the equivalent fractions to a common denominator and then comparing the magnitudes of their numerator. Thus, suppose $2 : 5$, $4 : 3$ and $4 : 5$ are three ratios to be compared

then the fractions $\frac{2}{5}$, $\frac{4}{3}$ and $\frac{4}{5}$ are reduced to equivalent fractions

with a common denominator. For this, the denominator of each is changed to 15 equal to the L.C.M. their denominators. Hence the

given ratios are expressed $\frac{6}{15}$, $\frac{20}{15}$ and $\frac{12}{15}$

or $2 : 5$, $4 : 3$, $4 : 5$ according to magnitude.

Example 1 Which of the ratios $2 : 3$ and $5 : 9$ is greater ?

Solution : In the form of fractions, the given ratios are $\frac{2}{3}$ and $\frac{5}{9}$,

Reducing them to fractions with a common denominator

they are written as $\frac{6}{9}$ and $\frac{5}{9}$.

Hence the greater ratio is $\frac{6}{9}$ or $2 : 3$.

Example 2 Are the ratios $3 : 4$ and $6 : 8$ equal ?

Solution The ratios are equal if $\frac{3}{4} = \frac{6}{8}$

These are equal if their cross products are equal; that is, if $3 \times 8 = 4 \times 6$. Since both of these products equal 24, the answer is yes, the ratios are equal.

REMEMBER

The two quantities must be of the same kind and in same unit.

The ratio is a pure number, i.e., without any unit of measurement.

The ratio would stay unaltered even if both the antecedent and the consequent are multiplied or divided by the same number

Compound Ratio : Ratios are compounded by multiplying together the antecedents for a new antecedent and the consequents for a new consequent.

The compound of $a : b$ and $c : d$ is

$$\frac{a \times c}{b \times d} \text{ i.e., } ac : bd.$$

Duplicate Ratio : The duplicate ratio of $x : y$ is $x^2 : y^2$.

Triplicate Ratio : The triplicate ratio of $x : y$ is $x^3 : y^3$.

Subduplicate Ratio : The subduplicate ratio of $x : y$ is $\sqrt{x} : \sqrt{y}$.

Subtriplicate Ratio : The subtriplicate ratio of $x : y$ is $\sqrt[3]{x} : \sqrt[3]{y}$.

Reciprocal Ratio : Reciprocal ratio of $a : b$ is

$$\frac{1}{a} : \frac{1}{b} \text{ or } b : a$$

Inverse Ratio : Inverse ratio of $x : y$ is $y : x$.

PROPERTIES

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$, i.e., the inverse ratios of two equal ratios are equal. The property is called **Invertendo**.

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$, i.e., the ratio of antecedents and consequents of two equal ratios are equal. This property is called **Alternendo**.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{b} = \frac{c+d}{d}.$$

This property is called **Componendo**.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a-b}{b} = \frac{c-d}{d}.$$

This property is called **Dividendo**.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{a-b} = \frac{c+d}{c-d}.$$

This property is called **Componendo – Dividendo**.

$$\text{If } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots. \text{ Then, Each ratio}$$

$$= \frac{\text{sum of Numerators}}{\text{sum of denominators}}$$

$$\text{i.e., } \frac{a}{b} = \frac{c}{d} = \frac{a+c+e+\dots}{b+d+f+\dots}$$

If we have two equations containing three unknowns as

$$a_1x + b_1y + c_1z = 0 \text{ and } \dots \dots \dots \text{ (i)}$$

$$a_2x + b_2y + c_2z = 0 \dots \dots \dots \text{ (ii)}$$

then, the values of x , y and z cannot be resolved without having a third equation.

However, in the absence of a third equation, we can find the proportion $x : y : z$.

This will be given by

$$b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1.$$

If the ratio between the first and the second quantities is $a : b$ and the ratio between the second and third quantities is $c : d$, then the ratio among first, second and third quantities is given by $ac : bc : bd$

If the ratio between the first and the second quantities is $a : b$ and the ratio between the second and third quantities is $c : d$ and the ratio between the third and fourth quantities is $e : f$, then the ratio among first, second, third and fourth quantities is given by $ace : bce : bde : bdf$

Ratio of the two variables of a homogeneous equation.

For this all the terms of the homogeneous equation are taken on one side and factorized into linear factors. A linear equation is formed from each of the factors and the ratio of the variables obtained.

Example 3 If $15(2x^2 - y^2) = 7xy$, then find the ratio $x : y$.

Solution : Given equation is $30x^2 - 7xy - 15y^2 = 0$

$$\text{or } (5x + 3y)(6x - 5y) = 0$$

$$\text{Hence, } 5x + 3y = 0 \text{ or } 6x - 5y = 0$$

$$\therefore \frac{x}{y} = \frac{-3}{5} \text{ or } \frac{5}{6}$$

$$\therefore x : y = -3 : 5 \text{ or } 5 : 6$$

Divide a Given Quantity into a Given Ratio.

Suppose any given quantity a , is to be divided in the ratio $m : n$.

Let one part of the given quantity be x then the other part will be $a - x$.

$$\therefore \frac{x}{a-x} = \frac{m}{n} \text{ or } nx = ma - mx$$

$$\text{or } (m+n)x = ma$$

$$\therefore \text{one part is } \frac{ma}{m+n} \text{ and the other part will be}$$

$$a - \frac{ma}{m+n} = \frac{na}{m+n}$$

Example 4 Divide 70 in the ratio 3 : 7.

Solution Let one part be x

$$\text{then the other part} = 70 - x$$

$$\therefore \frac{x}{70-x} = \frac{3}{7} \text{ or } 7x = 210 - 3x$$

$$\text{or } x = 21 \text{ and } 70 - x = 49$$

Hence the two required parts of 70 are 21 and 49.

If the ratio between two numbers is $a : b$ and if each number is increased by x , the ratio becomes $c : d$. Then

$$\text{Sum of the two numbers} = \frac{x(a+b)(c-d)}{ad-bc}$$

$$\text{Difference of the two numbers} = \frac{x(a-b)(c-d)}{ad-bc}$$

$$\text{Two numbers are given as } \frac{xa(c-d)}{ad-bc} \text{ and } \frac{xb(c-d)}{ad-bc}$$

Example 5 The ratio between two numbers is 3 : 4. If each number be increased by 2, the ratio becomes 7 : 9. Find the numbers.

$$\text{Solution : Numbers are } \frac{2 \times 3(7-9)}{3 \times 9 - 4 \times 7} \text{ and } \frac{2 \times 4(7-9)}{3 \times 9 - 4 \times 7}$$

$$\text{or } 12 \text{ and } 16.$$

If the sum of two numbers is A and their difference is a , then the ratio of numbers is given by $A + a : A - a$.

Example 6 The sum of two numbers is 60 and their difference is 6. What is the ratio of the two numbers?

Solution : The required ratio of the numbers

$$= \frac{60+6}{60-6} = \frac{66}{54} = \frac{11}{9} \text{ or } 11 : 9$$

PROPORTION

When two ratios are equal, the four quantities composing them are said to be in proportion.

If $\frac{a}{b} = \frac{c}{d}$, then a, b, c, d are in proportions.

This is expressed by saying that 'a' is to 'b' as 'c' is to 'd' and the proportion is written as

$$a : b :: c : d \quad \text{or} \quad a : b = c : d$$

The terms a and d are called the extremes while the terms b and c are called the means.

Mean Proportional.

If three quantities a, b and c are in continued proportion, then

$$a : b = b : c$$

$$\therefore ac = b^2$$

b is called mean proportional.

Example 7 Find the mean proportional between 3 and 75.

Solution : Let x be the required mean proportional. Then,

$$3 : x :: x : 75$$
$$\therefore x = \sqrt{3 \times 75} = 15$$

DIRECT AND INVERSE PROPORTION

Direct Proportion :

If on the increase of one quantity, the other quantity increases to the same extent or on the decrease of one, the other decreases to the same extent, then we say that the given two quantities are directly proportional. If A and B are directly proportional then we denote it by $A \propto B$.

$$\text{Also, } A = kB \text{ (k is constant)}$$

$$\Rightarrow \frac{A}{B} = k$$

If b_1 and b_2 are the values of B corresponding to the values a_1, a_2

of A respectively, then $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

Some Examples of direct proportion

- (i) Work done $\propto$ number of men
- (ii) Cost $\propto$ number of articles
- (iii) Work $\propto$ wages
- (iv) Working hour of a machine $\propto$ fuel consumed
- (v) Speed $\propto$ distance to be covered

Indirect Proportion (or inverse proportion)

If on the increase of one quantity, the other quantity decreases to the same extent or vice versa, then we say that the given two quantities are indirectly proportional. If A and

B are indirectly proportional then we denote it by $A \propto \frac{1}{B}$.

$$\text{Also, } A = \frac{k}{B} \text{ (k is a constant)} \Rightarrow AB = k$$

If b_1, b_2 are the values of B corresponding to the values a_1, a_2 of A respectively, then

$$a_1 b_1 = a_2 b_2$$

Some Examples of Indirect proportion:

$$\text{Number of men} \propto \frac{1}{\text{time}}$$

$$\text{Speed} \propto \frac{1}{\text{time to be covered distance}}$$

RULE OF THREE

In a problem on simple proportion, usually three terms are given and we have to find the fourth term, which we can solve by using Rule of three. In such problems, two of given terms are of same kind and the third term is of same kind as the required fourth term. First of all we have to find whether given problem is a case of direct proportion or indirect proportion. For this, write the given quantities under their respective headings and then mark the arrow in increasing direction. If both arrows are in same direction then the relation between them is direct otherwise it is indirect or inverse proportion. Proportion will be made by either head to tail or tail to head. The complete procedure can be understood by the examples.

Example 8 A man completes $\frac{5}{8}$ of a job in 10 days. At this rate, how many more days will it take him to finish the job ?

Solution : Work done = $\frac{5}{8}$. Remaining work = $\left(1 - \frac{5}{8}\right) = \frac{3}{8}$.

Work $\propto$ number of days (Direct Proportion)

Let the required number of days be x . Then,

Work	days
$\uparrow$ $\frac{5}{8}$	$\uparrow$ 10
$\uparrow$ $\frac{3}{8}$	$\uparrow$ x

$$\text{Then, } \frac{5}{8} : \frac{3}{8} :: 10 : x \Rightarrow \frac{5}{8} \times x = \frac{3}{8} \times 10$$

$$\Rightarrow x = \left(\frac{3}{8} \times 10 \times \frac{8}{5}\right) = 6 \text{ days}$$

Example 9 A fort had provision of food for 150 men for 45 days. After 10 days, 25 men left the fort. The number of days for which the remaining food will last, is :

Solution : After 10 days : 150 men had food for 35 days.

Suppose 125 men had food for x days. Now,

$$\text{Number of men} \propto \frac{1}{\text{number of days}} \text{ (Indirect Proportion)}$$

men	days
$\uparrow$ 150	$\downarrow$ 35
$\uparrow$ 125	$\downarrow$ x

Then, $125 : 150 :: 35 : x$

$$\Rightarrow 125 \times x = 150 \times 35$$
$$\Rightarrow x = \frac{150 \times 35}{125} \Rightarrow x = 42.$$

Hence, the remaining food will last for 42 days.

COMPOUND PROPORTION OR DOUBLE RULE OF THREE

In the compound proportion, number of ratios are more than two.

Example 10 If the cost of printing a book of 320 leaves with 21 lines on each page and on an average 11 words in each line is ₹ 19, find the cost of printing a book with 297 leaves, 28 lines on each page and 10 words in each line.

Solution: Number of leaves $\propto$ cost (Direct Proportion)
 Number of lines $\propto$ cost (Direct Proportion)
 Number of words $\propto$ cost (Direct Proportion)

$$\left. \begin{array}{l} \text{leaves} \quad 320 : 297 \\ \text{lines} \quad 21 : 28 \\ \text{words} \quad 11 : 10 \end{array} \right\} :: 19 : x$$

$$\therefore 320 \times 21 \times 11 \times x = 297 \times 28 \times 10 \times 19$$

$$\Rightarrow x = \frac{171}{8} = 21\frac{3}{8}$$

$$\text{Hence required cost} = ₹ 21\frac{3}{8}$$

PARTNERSHIP

A partnership is an association of two or more persons who invest their money in order to carry on a certain business.

A partner who manages the business is called the **working partner/active partner** and the one who simply invests the money is called the **sleeping partner**.

Partnership is of two kinds :

- (i) Simple (ii) Compound.

Simple partnership : If the capitals of the partners are invested for the same period, the partnership is called simple.

Compound partnership : If the capitals of the partners are invested for different lengths of time, the partnership is called compound.

If the period of investment is the same for each partner, then the profit or loss is divided in the ratio of their investments. If A and B are partners in a business, then

$$\frac{\text{Investment of } A}{\text{Investment of } B} = \frac{\text{Profit of } A}{\text{Profit of } B} \text{ or } = \frac{\text{Loss of } A}{\text{Loss of } B}$$

If A , B and C are partners in a business, then Investment of A : Investment of B : Investment of C is

Profit of A : Profit of B : Profit of C ,

Loss of A : Loss of B : Loss of C

Example 11 Three partner Rahul, Puneet and Chandan invest ₹ 1600, ₹ 1800 and ₹ 2300 respectively in a business. How should they divide a profit of ₹ 399 ?

Solution : Profit is to be divided in the ratio 16 : 18 : 23

$$\begin{aligned} \text{Rahul's share of profit} &= \frac{16}{16+18+23} \times 399 \\ &= \frac{16}{57} \times 399 = \text{Rs. } 112 \end{aligned}$$

$$\text{Puneet's share of profit} = \frac{18}{57} \times 399 = \text{Rs. } 126$$

$$\text{Chandan's share of profit} = \frac{23}{57} \times 399 = \text{Rs. } 161$$

Example 12 A , B and C enter into a partnership by investing 1500, 2500 and 3000 rupees, respectively. A as manager gets one-tenth of the total profit and the remaining profit is divided among the three in the ratio of their investment. If A 's total share is ₹ 369, find the shares of B and C .

Solution : If total profit is x , then

$$A's \text{ share} = \frac{1}{10}x + \frac{15}{15+25+30} \text{ of the balance } \frac{9}{10}x$$

$$\Rightarrow \frac{1}{10}x + \frac{27x}{140} = 369$$

$$\Rightarrow 14x + 27x = 369 \times 140$$

$$\Rightarrow x = \frac{369 \times 140}{41} = 9 \times 140 = 1260$$

$$B's \text{ share} = \frac{5}{14} \times \frac{9}{10} \times 1260 = ₹ 405$$

$$C's \text{ share} = \frac{6}{14} \times \frac{9}{10} \times 1260 = ₹ 486$$

Example 13 A and B invested in the ratio 3 : 2 in a business. If 5% of the total profit goes to charity and A 's share is ₹ 855, find the total profit.

Solution : Let the total profit be ₹ 100. Then, ₹ 5 goes to charity.

Now, ₹ 95 is divided in the ratio 3 : 2.

$$\therefore A's \text{ share} = \frac{95}{3+2} \times 3 = \text{Rs. } 57$$

But A 's actual share is ₹ 855.

$$\therefore \text{Actual total profit} = 855 \left(\frac{100}{57} \right) = ₹ 1500$$

MONTHLY EQUIVALENT INVESTMENT (MEI)

It is the product of the capital invested and the period for which it is invested.

If the period of investment is different, then the profit or loss is divided in the ratio of their Monthly Equivalent Investment.

$$\frac{\text{MEI } A}{\text{MEI } B} = \frac{\text{Profit of } A}{\text{Profit of } B} \text{ or } \frac{\text{Loss of } A}{\text{Loss of } B}$$

$$\text{i.e., } \frac{\text{Investment of } A \times \text{Period of Investment of } A}{\text{Investment of } B \times \text{Period of Investment of } B}$$

$$= \frac{\text{Profit of } A}{\text{Profit of } B} \text{ or } \frac{\text{Loss of } A}{\text{Loss of } B}$$

If A , B and C are partners in a business, then

MEI of A : MEI of B : MEI of C .

= Profit of A : Profit of B : Profit of C .

= Loss of A : Loss of B : Loss of C .

Example 14 A and B start a business. A invests ₹ 600 more than B and invest for 4 months and B for 5 months. A's share is ₹ 48 more than that of B, out of a total profit of ₹ 528. Find the capital contributed by each.

$$\text{Solution : } B's \text{ profit} = \frac{528 - 48}{2} = ₹ 240$$

$$A's \text{ profit} = 528 - 240 = ₹ 288$$

$$\frac{A's \text{ capital} \times 4}{B's \text{ capital} \times 5} = \frac{288}{240} = \frac{6}{5}$$

$$\therefore \frac{A's \text{ capital}}{B's \text{ capital}} = \frac{6}{5} \times \frac{5}{4} = \frac{3}{2}$$

$$\Rightarrow \frac{B's \text{ capital} + 600}{B's \text{ capital}} = \frac{3}{2}$$

$$\Rightarrow B's \text{ capital} = ₹ 1200 \text{ and } A's \text{ capital} = ₹ 1800$$

Example 15 Three persons A, B, C rent the grazing of a park for ₹ 570. A puts in 126 oxen in the park for 3 months, B puts in 162 oxen for 5 months and C puts in 216 oxen for 4 months. What part of the rent should each person pay?

$$\text{Solution : } \text{Monthly equivalent rent of } A = 126 \times 3 = 378$$

$$\text{Monthly equivalent rent of } B = 162 \times 5 = 810$$

$$\text{Monthly equivalent rent of } C = 216 \times 4 = 864$$

$$\therefore \text{Rent is to be divided in the ratio } 378 : 810 : 864, \text{ i.e., } 7 : 15 : 16$$

$$\therefore A \text{ would have to pay } \frac{7}{7+15+16} \text{ of the rent}$$

$$= \frac{7}{38} \text{ of the rent} = \frac{7}{38} \times 570 = ₹ 105$$

$$\therefore B \text{ would have to pay } \frac{15}{38} \text{ of the rent}$$

$$= \frac{15}{38} \times 570 = ₹ 225$$

$$\text{and } C \text{ would have to pay } \frac{16}{38},$$

$$\text{i.e. } \frac{8}{19} \text{ of the rent} = \frac{8}{19} \times 570 = ₹ 240$$

Example 16 Shekhar started a business investing ₹ 25,000 in 1999. In 2000, he invested an additional amount of ₹ 10,000 and Rajeev joined him with an amount of ₹ 35,000. In 2002, Shekhar invested another additional amount of ₹ 10,000 and Jatin joined them with an amount of ₹ 35,000. What will be Rajeev's share in the profit of ₹ 1,50,000 earned at the end of 3 years from the start of the business in 1999?

$$\begin{aligned} \text{Solution : Ratio of Shekhar, Rajeev and Jatin's investments} \\ &= 25000 \times 36 + 10000 \times 24 + 10000 \times 12 : 35000 \times 24 : \\ &35000 \times 12, \\ &= 25 \times 36 + 10 \times 24 + 10 \times 12 : 35 \times 24 : 35 \times 12 \\ &= 25 \times 3 + 10 \times 2 + 10 \times 1 : 35 \times 2 : 35 \times 1 \\ &= 75 + 20 + 10 : 70 : 35 \\ &= 105 : 70 : 35 \text{ i.e., } 3 : 2 : 1 \end{aligned}$$

$$\therefore \text{Rajeev's share in the profit} = \frac{2}{6} \times 150000 = ₹ 50000$$

Example 17 A began a business with ₹ 4500 and was joined afterwards by B with ₹ 5400. If the profits at the end of year was divided in the ratio 2 : 1 when did B join as a partner?

Solution : Let B joined after x months.

$$\text{Then, } 4500 \times 12 : 5400 \times (12 - x) = 2 : 1$$

$$\Rightarrow \frac{4500 \times 12}{5400 \times (12 - x)} = \frac{2}{1}$$

$$\Rightarrow \frac{45 \times 12}{54 \times (12 - x)} = \frac{2}{1}$$

$$\Rightarrow \frac{5}{12 - x} = 1$$

$$\Rightarrow x = 7 \text{ months}$$

MIXTURE (ALLIGATIONS)

Simple Mixture : When two different ingredients are mixed together, it is known as a simple mixture.

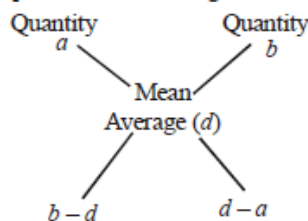
Compound Mixture : When two or more simple mixtures are mixed together to form another mixture, it is known as a compound mixture.

Alligation : Alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together. The word 'Alligation' literally means 'linking'.

Alligation rule : It states that when different quantities of the same or different ingredients of different costs are mixed together to produce a mixture of a mean cost, the ratio of their quantities is inversely proportional to the difference in their cost from the mean cost.

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Price of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{Price of Cheaper}}$$

Quicker representation of Alligation Rule :

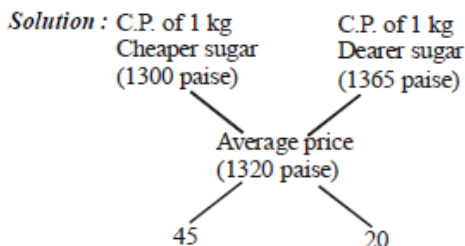


$$\frac{\text{Quantity of } a}{\text{Quantity of } b} = \frac{b-d}{d-a}$$

Applications of Alligation Rule :

- To find the mean value of a mixture when the prices of two or more ingredients, which are mixed together and the proportion in which they are mixed are given.
- To find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price.

Example 18 In what proportion must sugar at ₹ 13.00 per kg be mixed with sugar at ₹ 13.65 per kg, so that the mixture be worth ₹ 13.20 a kg ?

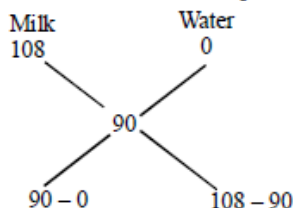


$$\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of dearer sugar}} = \frac{45}{20} = \frac{9}{4}$$

∴ They must be mixed in the ratio 9 : 4.

Example 19 A mixture of a certain quantity of milk with 16 litres of water is worth 90 P per litre. If pure milk be worth ₹ 1.08 per litre, how much milk is there in the mixture ?

Solution : The mean value is 90P and the price of water is 0 P.



By the Alligation Rule, milk and water are in the ratio of 5 : 1.

∴ Quantity of milk in the mixture = 5 × 16 = 80 litres.

Price of the Mixture :

When quantities Q_i of ingredients M_i 's with the cost C_i 's are mixed then cost of the mixture C_m is given by

$$C_m = \frac{\sum C_i Q_i}{\sum Q_i}$$

Example 20 5 kg of rice of ₹ 6 per kg is mixed with 4 kg of rice to get a mixture costing ₹ 7 per kg. Find the price of the costlier rice.

Solution : Let the price of the costlier rice be ₹ x.

By direct formula,

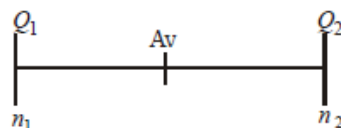
$$7 = \frac{6 \times 5 + 4 \times x}{9}$$

$$\Rightarrow 63 - 30 = 4x \Rightarrow 4x = 33$$

$$\Rightarrow x = \frac{33}{4} = 8.25$$

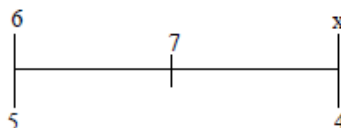
Straight line approach of Alligation

Let Q_1 and Q_2 be the two quantities, and n_1 and n_2 are the number of elements present in the two quantities respectively,



where Av is the average of the new group formed then n_1 corresponds to $Q_2 - \text{Av}$, n_2 corresponds to $\text{Av} - Q_1$ and $(n_1 + n_2)$ corresponds to $Q_2 - Q_1$.

Let us solve the previous example using straight line method



4 corresponds to 7 - 6 and 5 corresponds to x - 7.

i.e. 4 corresponds to 1

5 corresponds to 1.25

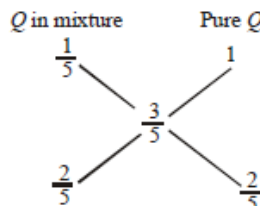
Hence, $x - 7 = 1.25$

$$\Rightarrow x = 8.25$$

Example 21 A jar contains a mixture of two liquids P and Q in the ratio 4 : 1. When 15 litres of the mixture is taken out and 15 litres of liquid Q is poured into the jar, the ratio becomes 2 : 3. How many litres of liquid P was contained in the jar.

$$\text{Solution : Fraction of } Q \text{ in original mixture} = \frac{1}{1+4} = \frac{1}{5}$$

$$\text{Fraction of } Q \text{ in resulting mixture} = \frac{3}{2+3} = \frac{3}{5}$$



Thus, the original mixture and liquid Q are mixed in the same ratio.

∴ If 15 litres of liquid Q is added, then after taking out 15 litres of mixture from the jar, there should have 15 litres of mixture left.

So, the quantity of mixture in the jar

$$= 15 + 15 = 30 \text{ litres}$$

$$\text{and quantity of P in the jar} = \frac{30}{5} \times 4 = 24 \text{ litres.}$$

Chapter 12: Time and Work

In most of the problems on time and work, either of the following basic parameters are to be calculated :

TIME :

If A can do a piece of work in X days, then A 's one day's work

$$= \frac{1}{X} \text{ th part of whole work.}$$

If A 's one day's work = $\frac{1}{X}$ th part of whole work, then A can finish the work in X days.

If A can do a piece of work in X days and B can do it in Y days then

A and B working together will do the same work in $\frac{XY}{X+Y}$ days.

If A , B and C can do a work in X , Y and Z days respectively then all of them working together can finish the work in

$$\frac{XYZ}{XY + YZ + XZ} \text{ days.}$$

Example 1 A can do a piece of work in 5 days, and B can do it in 6 days. How long will they take if both work together ?

Solution : A 's 1 day's work = $\frac{1}{5}$ th part of whole work and

B 's 1 day's work = $\frac{1}{6}$ th part of whole work

$\therefore (A + B)$'s one day's work = $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$ th part of

whole work. So, both together will finish the work in

$$\frac{30}{11} \text{ days} = 2\frac{8}{11} \text{ days.}$$

Quicker Method

$A + B$ can do the work in $\frac{5 \times 6}{5 + 6} \text{ days} = \frac{30}{11} = 2\frac{8}{11}$ days.

Example 2 Two men, Vikas and Vishal, working separately can mow a field in 8 and 12 hours respectively. If they work in stretches of one hour alternately, Vikas beginning at 8 a.m., when will the mowing be finished ?

Solution : In the first hour, Vikas mows $\frac{1}{8}$ of the field.

In the second hour, Vishal mows $\frac{1}{12}$ of the field.

$\therefore$ In the first 2 hours, $\left(\frac{1}{8} + \frac{1}{12} = \frac{5}{24}\right)$ of the field is mown.

$\therefore$ In 8 hours, $\frac{5}{24} \times 4 = \frac{5}{6}$ of the field is mown.

Now, $\left(1 - \frac{5}{6}\right) = \frac{1}{6}$ of the field remains to be mown.

In the 9th hour, Vikas mows $\frac{1}{8}$ of the field.

Remaining work = $\frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

$\therefore$ Vishal will finish the remaining work in $\left(\frac{1}{24} \div \frac{1}{12}\right)$ or $\frac{1}{2}$ of an hour.

$\therefore$ The total time required is $\left(8 + 1 + \frac{1}{2}\right)$ or $9\frac{1}{2}$ hours.

Thus, the work will be finished at $8 + 9\frac{1}{2} = 17\frac{1}{2}$ or 5.30 pm.

Example 3 A can do a piece of work in 36 days, B in 54 days and C in 72 days. All the three began the work together on the Dec. 15, 2012, but A left 8 days and B 12 days before the completion of the work. If C took the rest for a week then in how many days, the work was finished from the day it started ?

Solution : Let the total time taken be x days.
According to the given condition

$$\Rightarrow \frac{x-8}{36} + \frac{x-12}{54} + \frac{x}{72} = 1$$

$$\Rightarrow \frac{6(x-8) + 4(x-12) + 3x}{216} = 1$$

$$\Rightarrow \frac{6x - 48 + 4x - 48 + 3x}{216} = 1 \Rightarrow \frac{13x - 96}{216} = 1$$

$$\Rightarrow 13x - 96 = 216 \Rightarrow 13x = 216 + 96 = 312$$

$$\Rightarrow x = \frac{312}{13} = 24$$

Since, C takes the rest for a week, so the number of days in which the work was finished from one day it started = 31 i.e., on 14.01.2013.

Example 4 A and B can do a certain piece of work in 8 days, B and C can do it in 12 days and C and A can do it in 24 days. How long would each take separately to do it?

Solution : $(A+B)$'s one days' work = $1/18$,

$(A+C)$'s one days' work = $1/24$,

$(B+C)$'s one days' work = $1/12$,

Now add up all three equations :

$2(A+B+C)$'s one days' work

$$= \frac{1}{18} + \frac{1}{24} + \frac{1}{12} = \frac{13}{72}$$

$$(A+B+C)\text{'s one days' work} = \frac{13}{144}$$

A 's one days' work

= $(A+B+C)$'s one days' work - $(B+C)$'s one days' work

$$= \frac{13}{144} - \frac{1}{12} = \frac{1}{144}$$

Since A completes of the work in 1 day, he will complete work in

$$\frac{144}{1} = 144 \text{ days}$$

By similar logic we can find that B needs days and C will require

$$\frac{144}{5} \text{ days.}$$

If A and B together can do a piece of work in X days and A alone can do it in Y days, then B alone can do the work in

$$\frac{XY}{Y-X} \text{ days.}$$

Example 5 A and B together can do a piece of work in 6 days and A alone can do it in 9 days. In how many days can B alone do it?

Solution : $(A+B)$'s 1 day's work = $\frac{1}{6}$ th part of the whole work.

A 's 1 day's work = $\frac{1}{9}$ th part of the whole work.

$\therefore B$'s 1 day's work

$$= \frac{1}{6} - \frac{1}{9} = \frac{3-2}{18} = \frac{1}{18} \text{ th}$$

part of the whole work.

$\therefore B$ alone can do the work in 18 days.

Alternate Method

B alone can do the whole work in

$$\frac{6 \times 9}{9-6} = \frac{54}{3} = 18 \text{ days}$$

A and B can do a work in ' X ' and ' Y ' days respectively. They started the work together but A left ' a ' days before completion of the work. Then, time taken to finish the work is

$$\frac{Y(X+a)}{X+Y}$$

If ' A ' is ' a ' times efficient than B and A can finish a work in X days, then working together, they can finish the

work in $\frac{aX}{a+1}$ days.

If A is ' a ' times efficient than B and working together they finish a work in Z days then, time taken by

$$A = \frac{Z(a+1)}{a} \text{ days. and time taken by } B$$

$$= Z(a+1) \text{ days.}$$

If A working alone takes ' x ' days more than A and B together, and B working alone takes ' y ' days more than A and B together then the number of days taken by A and B working together is given by $[\sqrt{xy}]$ days.

Example 6 A and B can do alone a job in 6 days and 12 days.

They began the work together but 3 days before the completion of job, A leaves off. In how many days will the work be completed?

Solution : Let work will be completed in x days. Then, work done by A in $(x-3)$ days + work done by B in x days = 1

$$\text{i.e., } \frac{x-3}{6} + \frac{x}{12} = 1$$

$$\Rightarrow \frac{3x-6}{12} = 1 \Rightarrow x = 6 \text{ days}$$

Alternate Method

$$\text{Required time} = \frac{12(6+3)}{12+6} = 6 \text{ days}$$

Example 7 A is half good a workman as B and together they finish a job in 14 days. In how many days working alone will B finish the job?

Solution : Let B can do the work in x days
and A can do the work in $2x$ days

$$\text{Then, } \frac{1}{x} + \frac{1}{2x} = \frac{1}{14}$$

$$\Rightarrow x = \frac{3}{2} \times 14 = 21 \text{ days}$$

Alternate Method

$$\text{Time taken by B} = 14 \left(\frac{1}{2} + 1 \right) = 21 \text{ days}$$

If n men or m women can do a piece of work in X days, then
 N men and M women together can finish the work in

$$\frac{nmX}{nM + mN} \text{ days.}$$

Example 8 10 men can finish a piece of work in 10 days, where
as it takes 12 women to finish it in 10 days. If 15 men and 6
women undertake to complete the work, how many days they will
take to complete it?

Solution : It is clear that 10 men = 12 women or 5 men = 6
women

$$\Rightarrow 15 \text{ men} + 6 \text{ women} = (18 + 6) \text{ i.e., } 24 \text{ women}$$

Now 12 women can complete the work in 10 days

$$\therefore 24 \text{ women will do it in } 5 \text{ days.}$$

Alternate Method

$$\text{Required time} = \frac{10 \times 12 \times 10}{10 \times 6 + 12 \times 15} = 5 \text{ days}$$

Example 9 If 3 men or 4 women can reap a field in 43 days, how
long will 7 men and 5 women take to reap it?

Solution : 3 men reap $\frac{1}{43}$ of the field in 1 day.

$$\therefore 1 \text{ men reaps } \frac{1}{43 \times 3} \text{ of the field in 1 day.}$$

$$4 \text{ women reap } \frac{1}{43} \text{ of the field in 1 day.}$$

$$\therefore 1 \text{ woman reaps } \frac{1}{43 \times 4} \text{ of the field in 1 day.}$$

$$\therefore 7 \text{ men and } 5 \text{ women reap } \left(\frac{7}{43 \times 3} + \frac{5}{43 \times 4} \right)$$

$$= \frac{1}{12} \text{ of the field in 1 day.}$$

$$\therefore 7 \text{ men and } 5 \text{ women will reap the whole field in } 12 \text{ days.}$$

Alternate Method

$$\begin{aligned} \text{Required number of days} &= \frac{1}{\left[\frac{7}{43 \times 3} + \frac{5}{43 \times 4} \right]} \\ &= \frac{43 \times 3 \times 4}{4 \times 4 + 5 \times 3} = 12 \text{ days.} \end{aligned}$$

Example 10 If 12 men and 16 boys can do a piece of work in 5
days and 13 men and 24 boys can do it in 4 days, how long will 7
men and 10 boys take to do it?

Solution : 12 m and 16 b can do the work in 5 days (i)

$$13 \text{ m and } 24 \text{ b can do the work in } 4 \text{ days (ii)}$$

Now it is easy to see that if the number of workers be
multiplied by any number, the time must be divided by the
same number (derived from : more worker less time).

Hence multiplying the number of workers in (i) and (ii) by
5 and 4 respectively, we get 5 (12 m + 16 b) can do the work
in $5/5 = 1$ day

$$4 (13 \text{ m} + 24 \text{ b}) \text{ can do the work in } \frac{4}{4} = 1 \text{ day}$$

$$\Rightarrow 5(12\text{m} + 16\text{b}) = 4(13\text{m} + 24\text{b})$$

$$\Rightarrow 60\text{m} + 80\text{b} = 52\text{m} + 96\text{b}$$

$$\Rightarrow 60\text{m} - 52\text{m} = 96\text{b} - 80\text{b}$$

$$\Rightarrow 8\text{m} = 16\text{b}$$

$$\therefore 1\text{m} = 2\text{b.}$$

$$\text{Thus, } 12\text{m} + 16\text{b} = 24\text{b} + 16\text{b} = 40\text{b}$$

$$\text{and } 13\text{m} + 24\text{b} = 26\text{b} + 24\text{b} = 50\text{b}$$

The question now becomes :

"If 40 boys can do a piece of work in 6 days how long will
24 boys take to do it?"

Using basic formula, we have

$$40 \times 5 = 24 \times D_2$$

$$\Rightarrow D_2 = \frac{40 \times 5}{24} = 8 \frac{1}{3} \text{ days}$$

Example 11 Kaberi takes twice as much time as Kanti and thrice
as much as Kalpana to finish a piece of work. They together
finish the work in one day. Find the time taken by each of them to
finish the work.

Solution : Here, the alone time of Kaberi is related to the alone
times of other two persons, so assume the alone time of
Kaberi = x ,

$$\text{Then, alone time of Kanti} = \frac{x}{2} \text{ and of Kalpana} = \frac{x}{3}$$

Kaberi's 1 day work + Kanti's 1 day work + Kalpana's
1 day work = combined 1 days work

$$\Rightarrow \frac{1}{x} + \frac{1}{x/2} + \frac{1}{x/3} = \frac{1}{1} \Rightarrow x = 6$$

$\therefore$ Alone time for Kaberi = 6 days, for Kanti = $6/2 = 3$
days, Kalpana = $6/3 = 2$ days,

Example 12 1 man or 2 women or 3 boys can do a work in 44 days.
Then in how many days will 1 man, 1 woman and 1 boy do the work?

Solution : Number of required days

$$= \frac{1}{\frac{1}{44 \times 1} + \frac{1}{44 \times 2} + \frac{1}{44 \times 3}} = \frac{44 \times 1 \times 2 \times 3}{6 + 3 + 2} = 24 \text{ days}$$

If ' M_1 ' persons can do ' W_1 ' works in ' D_1 ' days and ' M_2 ' persons can do ' W_2 ' works in ' D_2 ' days then

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

If T_1 and T_2 are the working hours for the two groups then $M_1 D_1 W_2 T_1 = M_2 D_2 W_1 T_2$

Similarly,

$$M_1 D_1 W_2 T_1 E_1 = M_2 D_2 W_1 T_2 E_2$$

where E_1 and E_2 are the efficiencies of the two groups.

If the number of men to do a job is changed in the ratio $a : b$, then the time required to do the work will be in the ratio $b : a$, assuming the amount of work done by each of them in the given time is the same, or they are identical.

A is K times as good a worker as B and takes X days less than B to finish the work. Then the amount of time required by A and B working together is

$$\frac{K \times X}{K^2 - 1} \text{ days.}$$

If A is n times as efficient as B , i.e. A has n times as much capacity to do work as B , A will take $\frac{1}{n}$ of the time taken by B to do the same amount of work.

Example 13 5 men prepare 10 toys in 6 days working 6 hrs a day. Then in how many days can 12 men prepare 16 toys working 8 hrs a day?

Solution : This example has an extra variable 'time' (hrs a day), so the 'basic-formula' can't work in this case. An extended formula is being given :

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$$

Here, $5 \times 6 \times 6 \times 16 = 12 \times D_2 \times 8 \times 10$

$\therefore$

$$D_2 = \frac{5 \times 6 \times 6 \times 16}{12 \times 8 \times 10} = 3 \text{ days.}$$

Example 14 A and B can do a work in 45 days and 40 days respectively. They began the work together, but A left after some time and B finished the remaining work in 23 days. After how many days did A leave?

Solution : B works alone for 23 days.

$$\therefore \text{Work done by } B \text{ in 23 days} = \frac{23}{40} \text{ work}$$

$$\therefore A + B \text{ together } 1 - \frac{23}{40} = \frac{17}{40} \text{ work}$$

$$\text{Now, } A + B \text{ do 1 work in } \frac{40 \times 45}{40 + 45} = \frac{40 \times 45}{85} \text{ days}$$

$$\therefore A + B \text{ do } \frac{17}{40} \text{ work in } \frac{40 \times 45}{85} \times \frac{17}{40} = 9 \text{ days.}$$

Alternate method

If we ignore the intermediate steps, we can write a

$$\text{direct formula as : } \frac{40 \times 45}{40 + 45} \left(\frac{40 - 23}{40} \right) = 9 \text{ days.}$$

Example 15 Two friends take a piece of work for Rs 960. One alone could do it in 12 days, the other in 16 days with the assistance of an expert they finish it in 4 days. How much remuneration the expert should get?

Solution : First friend's 4 day's work = $\frac{4}{12} = \frac{1}{3}$ (Since, the work is finished in 4 days, when expert assists)

$$\text{Second friend's 4 day's work} = \frac{4}{16} = \frac{1}{4}$$

$$\text{The expert's 4 day's work} = 1 - \left(\frac{1}{3} + \frac{1}{4} \right) = \frac{5}{12}$$

Now, total wages of Rs 960 is to be distributed among two friends and the expert in proportion to the amount of work done by each of them.

So, 960 is to be divided in the proportion of

$$\frac{1}{3} : \frac{1}{4} : \frac{5}{12} \text{ or } 4 : 3 : 5$$

$$\therefore \text{Share of expert} = \frac{5}{12} \times 960 = ₹ 400$$

Hence, the expert should get ₹ 400.

WORK AND WAGES

Wages are distributed in proportion to the work done and in indirect proportion to the time taken by the individual.

Example 16 A , B and C can do a work in 6, 8 and 12 days respectively. Doing that work together they get an amount of ₹ 1350. What is the share of B in that amount?

$$\text{Solution : } A's \text{ one day's work} = \frac{1}{6}$$

$$B's \text{ one day's work} = \frac{1}{8}$$

$$C's \text{ one day's work} = \frac{1}{12}$$

$$A's \text{ share} : B's \text{ share} : C's \text{ share}$$

$$= \frac{1}{6} : \frac{1}{8} : \frac{1}{12}$$

Multiplying each ratio by the L.C.M. of their denominators, the ratios become $4 : 3 : 2$

$$\therefore B's \text{ share} = \frac{1350 \times 3}{9} = ₹ 450$$

Example 17 If 6 men working 8 hours a day earn ₹1680 per week, then how much will 9 men working 6 hours a day earn per week?

Solution :

6m	8 hours	₹ 1680
9m	6 hours	?

$$? = 1680 \times \frac{6}{8} \times \frac{9}{6} = ₹ 1890$$

Alternate method

As earnings are proportional to the work done, we have

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2} \Rightarrow \frac{6 \times 8}{1680} = \frac{9 \times 6}{W_2} \Rightarrow W_2 = ₹ 1890$$

Chapter 13: Pipes and Cisterns

PIPES AND CISTERNS

The same principle of Time and Work is employed to solve the problems on Pipes and Cisterns. The only difference is that in this case, the work done is in terms of filling or emptying a cistern (tank) and the time taken is the time taken by a pipe or a leak (crack) to fill or empty a cistern respectively.

Inlet : A pipe connected with a tank (or a cistern or a reservoir) is called an inlet, if it fills it.

Outlet : A pipe connected with a tank is called an outlet, if it empties it.

If a pipe can fill a tank in x hours, then the part filled in 1 hour

$$= \frac{1}{x}$$

If a pipe can empty a tank in y hours, then the part of the full tank emptied in 1 hour

$$= \frac{1}{y}$$

If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened

$$= \left(\frac{1}{x} - \frac{1}{y} \right)$$

$\therefore$ Time taken to fill the tank, when both the pipes are opened

$$= \frac{xy}{y-x}$$

If a pipe can fill a tank in x hours and another can fill the same tank in y hours, then time taken to fill the tank when both the pipes are opened

$$= \frac{xy}{y+x}$$

If a pipe fills a tank in x hours and another fills the same tank in y hours, but a third one empties the full tank in z hours, and all of them are opened together, then net part filled in 1 hr

$$= \left[\frac{1}{x} + \frac{1}{y} - \frac{1}{z} \right]$$

$\therefore$ Time taken to fill the tank = $\frac{xyz}{yz + xz - xy}$ hours.

A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank

$$= \frac{xy}{y-x} \text{ hrs.}$$

A cistern has a leak which can empty it in X hours. A pipe which admits Y litres of water per hour into the cistern is

turned on and now the cistern is emptied in Z hours. Then the capacity of the cistern

$$= \frac{X+Y+Z}{Z-X} \text{ litres.}$$

A cistern is filled by three pipes whose diameters are X cm., Y cm. and Z cm. respectively (where $X < Y < Z$). Three pipes are running together. If the largest pipe alone will fill it in P minutes and the amount of water flowing in by each pipe is proportional to the square of its diameter, then the time in which the cistern will be filled by the three pipes is

$$\left[\frac{PZ^2}{X^2 + Y^2 + Z^2} \right] \text{ minutes.}$$

If one filling pipe A is n times faster and takes X minutes less time than the other filling pipe B , then the time they will take to fill a cistern, if both the pipes are opened together, is

$$\left[\frac{nX}{(n^2 - 1)} \right] \text{ minutes.}$$

A will fill the cistern in $\left(\frac{X}{n-1} \right)$ minutes and

B will take to fill the cistern $\left(\frac{nX}{n-1} \right)$ minutes.

Here, A is the faster filling pipe and B is the slower one.

Two filling pipes A and B opened together can fill a cistern in t minutes. If the first filling pipe A alone takes X minutes more or less than t and the second fill pipe B alone takes Y minutes more or less than t minutes, then t is given by

$$[t = \sqrt{XY}] \text{ minutes.}$$

Example 1 A pipe can fill a cistern in 6 hours. Due to a leak in its bottom, it is filled in 7 hours. When the cistern is full, in how much time will it be emptied by the leak?

Solution : Part of the capacity of the cistern emptied by the

$$\text{leak in one hour} = \left(\frac{1}{6} - \frac{1}{7} \right) = \frac{1}{42} \text{ of the cistern.}$$

The whole cistern will be emptied in 42 hours.

Example 2 Three pipes A , B and C can fill a cistern in 6 hrs. After working together for 2 hrs, C is closed and A and B fill the cistern in 8 hrs. Then find the time in which the cistern can be filled by pipe C .

Solution : $A + B + C$ can fill in 1 hr = $\frac{1}{6}$ of cistern.

$A + B + C$ can fill in 2 hrs $= \frac{2}{6} = \frac{1}{3}$ of cistern.

Remaining part $= \left(1 - \frac{1}{3}\right) = \frac{2}{3}$ is filled by $A + B$ in 8 hrs.

$\therefore (A + B)$ can fill the cistern in $\frac{8 \times 3}{2} = 12$ hrs.

Since $(A + B + C)$ can fill the cistern in 6 hrs.

$\therefore C = (A + B + C) - (A + B)$ can fill the cistern in

$$\frac{12 \times 6}{12 - 6} \text{ hours} = 12 \text{ hours.}$$

Example 3 Pipe A can fill a tank in 20 hours while pipe B alone can fill it in 30 hours and pipe C can empty the full tank in 40 hours. If all the pipes are opened together, how much time will be needed to make the tank full?

Solution: By direct formula,

$$\begin{aligned} \text{Required time} &= \frac{20 \times 30 \times 40}{30 \times 40 + 20 \times 40 - 20 \times 30} \\ &= \frac{120}{7} = 17\frac{1}{7} \text{ hrs.} \end{aligned}$$

Example 4 Three pipes A , B and C can fill a tank in 6 minutes, 8 minutes and 12 minutes, respectively. The pipe C is closed 6 minutes before the tank is filled. In what time will the tank be full?

Solution: Let it takes t minutes to completely fill the tank.

$$\text{Now, } \frac{t}{6} + \frac{t}{8} + \frac{t-6}{12} = 1$$

$$\Rightarrow \frac{4t + 3t + 2t - 12}{24} = 1$$

$$\Rightarrow 9t - 12 = 24$$

$$\Rightarrow 9t = 36 \Rightarrow t = 4 \text{ min.}$$

Example 5 If three taps are opened together, a tank is filled in 12 hrs. One of the taps can fill it in 10 hrs and another in 15 hrs. How does the third tap work?

Solution: We have to find the nature of the third tap, whether it is a filler or a waste pipe.

Let it be a filler pipe which fills in x hrs.

$$\text{Then, } \frac{10 \times 15 \times x}{10 \times 15 + 10x + 15x} = 12$$

$$\Rightarrow 150x = 150 \times 12 + 25x \times 12$$

$$\Rightarrow -150x = 1800$$

$$\therefore x = -12$$

-ve sign shows that the third pipe is a waste pipe which vacates the tank in 12 hrs.

Example 6 A and B can fill a cistern in 7.5 minutes and 5 minutes respectively and C can carry off 14 litres per minute. If the cistern is already full and all the three pipes are opened, then it is emptied in 1 hour. How many litres can it hold?

Solution: If the capacity is L litres, water filled in 1 hour
= Water removed in 1 hour.

$$L + \frac{L}{7\frac{1}{2}} \times 60 + \frac{L}{5} \times 60 = 14 \times 60$$

$$\therefore L + \frac{2L}{15} \times 60 + 12L = 14 \times 60$$

$$\Rightarrow L + 8L + 12L = 14 \times 60$$

$$\Rightarrow 21L = 14 \times 60 \text{ or } L = 40 \text{ litres.}$$

So the capacity of the cistern is 40 litres.

- Now, it is the turn of A and B and $\frac{3}{20}$ part is filled by A and B in 1 hour.
 $\therefore$ Total time taken to fill the tank = (6 + 1) hrs = 7 hrs.
23. (c) Work done by the waste pipe in 1 minute

$$= \frac{1}{15} - \left(\frac{1}{20} + \frac{1}{24} \right) = \left(\frac{1}{15} - \frac{11}{120} \right) = -\frac{1}{40}$$
 [-ve sign means emptying]
 $\therefore$ Volume of $\frac{1}{40}$ part = 3 gallons.
 Volume of whole = (3 $\times$ 40) gallons = 120 gallons.
24. (b) Let x kg of oil is used for eating purpose, daily, then
 $(x + 11) \times 50 = (x + 15) \times 45$
 $x = 25$
 $\therefore$ Total quantity of oil = (25 + 11) $\times$ 50 = 1800
 $\therefore$ Required number of days = $\frac{1800}{25} = 72$ days
25. (b) Part filled by (A + B + C) in 3 minutes

$$= 3 \left(\frac{1}{30} + \frac{1}{20} + \frac{1}{10} \right) = \left(3 \times \frac{11}{60} \right) = \frac{11}{20}$$

 Part filled by C in 3 minutes = $\frac{3}{10}$
 $\therefore$ Required ratio = $\left(\frac{3}{10} \times \frac{20}{11} \right) = \frac{6}{11}$
26. (b) Part of tank filled in 1 hour = $\frac{1}{3}$
 Part of tank emptied in the same time = $\frac{1}{3} - \frac{1}{3.5}$
 Total time required to empty it = $\frac{1}{\frac{1}{3} - \frac{1}{3.5}} = 21$ hours
27. (d) Pipe 1 (Hot) $\rightarrow 3 + X$, $X \rightarrow$ Pipe 2 (cold)
 Together $\frac{X(X+3)}{2X+3} = 6\frac{2}{3}$ min.

$$= \frac{X(X+3)}{2X+3} = 6\frac{2}{3} \text{ min.} = \frac{20}{3}$$

 $40X + 60 = 3X(X+3)$
 $\Rightarrow 40X + 60 = 3X^2 + 9X$
 $\Rightarrow 3X^2 - 31X - 60 = 0$
 $\Rightarrow X = 12$ minutes
28. (a) Let A and B together work for x minutes then amount of water filled in the period = $x \left(\frac{1}{30} + \frac{1}{40} \right) = \frac{7x}{120}$
 Remaining part = $1 - \frac{7x}{120} = \left(\frac{120 - 7x}{120} \right)$
 Work done by A in (10 - x) minutes = $\frac{120 - 7x}{120}$

- $$\frac{7x}{120} + \frac{10 - x}{30} = 1 \text{ or } 7x + 40 - 4x = 120$$
- $$3x = 120 - 40 = 80$$
- $$x = 26\frac{2}{3} \text{ min}$$
29. (c) Let h be the length of water column discharged in 1 hour or 1 minute.
 Volume discharged by the 4 pipes = Volume discharged by the single pipe.
 $4 \times \pi \times (1.5)^2 \times h = \pi \times (r)^2 \times h$
 $r^2 = 9$ $\therefore r = 3$, Diameter = 6 cm.
30. (c) Let the cistern be filled in T min.
 $\therefore$ Pipe A is opened for T min and pipe B is opened for 3 min.
 Using the formula, for filling the cistern,
 A's amount of work + B's amount of work = 1.

$$\Rightarrow \frac{T}{10} + \frac{3}{15} = 1 \Rightarrow T = 10 \left(1 - \frac{1}{5} \right) = 8.$$

 $\therefore$ The cistern is filled in 8 min.
31. (c) Part filled by the inlet pipe in 1 hour

$$= \left(\frac{1}{10} - \frac{1}{16} \right) = \frac{6}{160} = \frac{3}{80}$$

 Part filled by the inlet pipe in 1 minute

$$= \frac{3}{80 \times 60} = \frac{1}{1600}$$

 $\therefore$ Capacity of tank = $1600 \times 8 = 12800$ litres.
32. (d) Suppose the first pipe was closed after x hrs.
 Then, first's x hrs' supply + second's 16 hrs' supply = 1
 Or, $\frac{x}{24} + \frac{16}{32} = 1 \therefore \frac{x}{24} = 1 - \frac{1}{2} = \frac{1}{2} \therefore x = 12$ hrs.
 Alternate method :
 The first pipe should work for $\left(1 - \frac{16}{32} \right) \times 24$ hrs. = 12 hrs
33. (b) Let the faster pipe fills the tank in x hrs.
 Then the slower pipe fills the tank in x + 10 hrs.
 When both of them are opened, the reservoir will be filled in $\frac{x(x+10)}{x+(x+10)} = 12$
 Or, $x^2 - 14x - 120 = 0 \therefore x = 20, -6$
 But x can't be -ve, hence the faster pipe will fill the reservoir in 20 hrs.
34. (a) It is clear from the question that the filler pipe fills the tank in 8 hrs and if both the filler and the leak work together, the tank is filled in 8 hrs. Therefore the leak will empty the tank in $\frac{8 \times 10}{10 - 8} = 40$ hrs.

Chapter 14: Speed, Time and Distance

SPEED

The rate at which any moving body covers a particular distance is called its speed.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}; \text{Time} = \frac{\text{Distance}}{\text{Speed}};$$

Distance = Speed $\times$ time

Unit : SI unit of speed is metre per second (mps). It is also measured in kilometers per hour (kmph) or miles per hour (mph).

Conversion of units :

$$1 \text{ hour} = 60 \text{ minutes} = 60 \times 60 \text{ seconds.}$$

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ km} = 0.625 \text{ mile}$$

$$\Rightarrow 1 \text{ mile} = 1.60 \text{ km, i.e.,}$$

$$8 \text{ km} = 5 \text{ miles}$$

$$1 \text{ yard} = 3 \text{ feet}$$

$$1 \text{ foot} = 12 \text{ inches}$$

$$1 \text{ km/h} = \frac{5}{18} \text{ m/sec,}$$

$$1 \text{ m/sec} = \frac{18}{5} \text{ km/h}$$

$$1 \text{ miles/hr} = \frac{22}{15} \text{ ft/sec}$$

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total time}}$$

While travelling a certain distance d , if a man changes his speed in the ratio $m : n$, then the ratio of time taken becomes $n : m$.

If a certain distance (d), say from A to B , is covered at ' a ' km/hr and the same distance is covered again say from B to A in ' b ' km/hr, then the average speed during the whole journey is given by :

$$\text{Average speed} = \left(\frac{2ab}{a+b} \right) \frac{\text{km}}{\text{hr}}$$

... (which is the harmonic means of a and b)

Also, if t_1 and t_2 is time taken to travel from A to B and B to A , respectively, the distance ' d ' from A to B is given by :

$$d = (t_1 + t_2) \left(\frac{ab}{a+b} \right)$$

$$d = (t_1 - t_2) \left(\frac{ab}{b-a} \right)$$

$$d = (a - b) \left(\frac{t_1 t_2}{t_2 - t_1} \right)$$

If a body travels a distance ' d ' from A to B with speed ' a ' in time t_1 and travels back from B to A i.e., the same distance

with $\frac{m}{n}$ of the usual speed ' a ', then the change in time taken to cover the same distance is given by :

$$\text{Change in time} = \left(\frac{m}{n} - 1 \right) \times t_1; \text{ for } n > m$$

$$= \left(1 - \frac{n}{m} \right) \times t_1; \text{ for } m > n$$

If first part of the distance is covered at the rate of v_1 in time t_1 and the second part of the distance is covered at the rate of v_2 in time t_2 , then the average speed is

$$\left(\frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} \right)$$

Relative Speed : When two bodies are moving in same direction with speeds S_1 and S_2 respectively, their relative speed is the difference of their speeds.

$$\text{i.e. Relative Speed} = S_1 - S_2$$

When two bodies are moving in opposite direction with speeds S_1 and S_2 respectively, their relative speed is the sum of their speeds.

$$\text{i.e. Relative Speed} = S_1 + S_2$$

Example 1 The driver of a maruti car driving at the speed of 68 km/h locates a bus 40 metres ahead of him. After 10 seconds, the bus is 60 metres behind. Find the speed of bus.

Solution : Let speed of Bus = S_B km/h.

$$\begin{aligned} \text{Now, in 10 sec., car covers the relative distance} \\ = (60 + 40) \text{ m} = 100 \text{ m} \end{aligned}$$

$$\therefore \text{Relative speed of car} = \frac{100}{10} = 10 \text{ m/s}$$

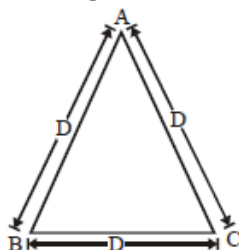
$$= 10 \times \frac{18}{5} = 36 \text{ km/h}$$

$$\therefore 68 - S_B = 36$$

$$\Rightarrow S_B = 32 \text{ km/h}$$

Example 2 If a person goes around an equilateral triangle shaped field at speed of 10, 20 and 40 kmph on the first, second and third side respectively and reaches back to the starting point, then find his average speed during the journey.

Solution : Let the measure of each side of triangle is D km. The person travelled the distance from A to B with 10 kmph, B to C with 20 kmph and C to A with 40 kmph.



If T_{AB} = Time taken by the person to travel from A to B ,
 T_{BC} = Time taken by the person to travel from B to C
 and T_{CA} = Time taken by the person to travel from C to A .

Then total time = $T_{AB} + T_{BC} + T_{CA}$

$$= \frac{D}{10} + \frac{D}{20} + \frac{D}{40} = D \left(\frac{8+4+2}{80} \right) = \frac{7D}{40}$$

Total distance travelled = $D + D + D = 3D$

Hence, average speed

$$= \frac{3D}{\frac{7D}{40}} = \frac{120}{7} = 17\frac{1}{7} \text{ kmph.}$$

Example 3 Two guns were fired from the same place at an interval of 15 min, but a person in a bus approaching the place hears the second report 14 min 30 sec after the first. Find the speed of the bus, supposing that sound travels 330 m per sec.

Solution : Distance travelled by the bus in 14 min 30 sec could be travelled by sound in (15 min – 14 min 30 sec) = 30 second.

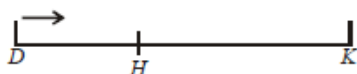
$$\therefore \text{Bus travels} = 330 \times 30 \text{ in } 14\frac{1}{2} \text{ min.}$$

$\therefore$ Speed of the bus per hour

$$= \frac{330 \times 30 \times 2 \times 60}{29 \times 1000} = \frac{99 \times 12}{29} = \frac{1188}{29} = 40\frac{28}{29} \text{ km/hr}$$

Example 4 A hare sees a dog 100 m away from her and scuds off in the opposite direction at a speed of 12 km/h. A minute later the dog perceives her and gives chase at a speed of 16 km/h. How soon will the dog overtake the hare and at what distance from the spot where the hare took flight?

Solution : Suppose the hare at H sees the dog at D .



$$\therefore DH = 100 \text{ m}$$

Let K be the position of the hare where the dog sees her.

$\therefore HK$ = the distance gone by the hare in 1 min

$$= \frac{12 \times 1000}{60} \times 1 \text{ m} = 200 \text{ m}$$

$$\therefore DK = 100 + 200 = 300 \text{ m}$$

The hare thus has a start of 300 m.

Now the dog gains (16 – 12) or 4 km/h.

$\therefore$ The dog will gain 300 m in

$$\frac{60 \times 300}{4 \times 1000} \text{ min or } 4\frac{1}{2} \text{ min.}$$

Again, the distance gone by the hare in $4\frac{1}{2}$ min

$$= \frac{12 \times 1000}{60} \times 4\frac{1}{2} = 900 \text{ m}$$

$\therefore$ Distance of the place where the hare is caught from the spot H where the hare took flight = $200 + 900 = 1100 \text{ m}$

If two persons (or vehicles or trains) start at the same time in opposite directions from two points A and B , and after crossing each other they take x and y hours respectively to complete the journey, then

$$\frac{\text{Speed of first}}{\text{Speed of second}} = \sqrt{\frac{y}{x}}$$

Example 5 A train starts from A to B and another from B to A at the same time. After crossing each other they complete their journey in $3\frac{1}{2}$ and $2\frac{4}{7}$ hours respectively. If the speed of the first is 60 km/h, then find the speed of the second train.

$$\text{Solution : } \frac{\text{1st train's speed}}{\text{2nd train's speed}} = \sqrt{\frac{y}{x}} = \sqrt{\frac{2\frac{4}{7}}{3\frac{1}{2}}}$$

$$= \sqrt{\frac{18}{7} \times \frac{2}{7}} = \frac{6}{7}$$

$$\therefore \frac{60}{\text{2nd train's speed}} = \frac{6}{7}$$

$$\Rightarrow \text{2nd train's speed} = 70 \text{ km/h.}$$

If new speed is $\frac{a}{b}$ of usual speed, then

$$\text{Usual time} = \frac{\text{Change in time}}{\left(\frac{b}{a} - 1\right)}$$

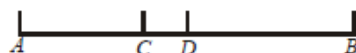
Example 6 A boy walking at $\frac{3}{5}$ of his usual speed, reaches his school 14 min late. Find his usual time to reach the school.

$$\text{Solution : Usual time} = \frac{14}{\frac{5}{3} - 1} = \frac{14 \times 3}{2} = 21 \text{ min}$$

Example 7 A train after travelling 50 km, meets with an accident

and then proceeds at $\frac{4}{5}$ of its former rate and arrives at the terminal 45 minutes late. Had the accident happened 20 km further on, it would have arrived 12 minutes sooner. Find the speed of the train and the distance.

Solution : Let A be the starting place, B the terminal, C and D the places where the accidents to be placed.



By travelling at $\frac{4}{5}$ of its original rate the train would

take $\frac{5}{4}$ of its usual time, i.e., $\frac{1}{4}$ of its original time more.

$\therefore \frac{1}{4}$ of the usual time taken to travel the distance

$$CB = 45 \text{ min.} \quad \dots(i)$$

and $\frac{1}{4}$ of the usual time taken to travel the distance

$$DB = (45 - 12) \text{ min} \quad \dots(ii)$$

Subtracting (ii) from (i),

$\frac{1}{4}$ of the usual time taken to travel the distance

$$CD = 12 \text{ min.}$$

$\therefore$ Usual time taken on travel 20 km = 48 min.

$\therefore$ Speed of the train per hour = $\frac{20}{48} \times 60$ or 25 km/h.

From (i), we have

Time taken to travel $CB = 45 \times 4 \text{ min} = 3 \text{ hrs.}$

$\therefore$ The distance $CB = 25 \times 3$ or 75 km.

Hence the distance $AB =$ the distance $(AC + CB)$
 $= 50 + 75$ or 125 km.

A man covers a certain distance D . If he moves S_1 speed faster, he would have taken t time less and if he moves S_2 speed slower, he would have taken t time more. The original speed is given by

$$\frac{2 \times (S_1 \times S_2)}{S_2 - S_1}$$

Example 8 A man covers a certain distance on scooter. Had he moved 3 km/h faster, he would have taken 20 min less. If he had moved 2 km/h slower, he would have taken 20 min more. Find the original speed.

Solution : Speed = $\frac{2 \times (3 \times 2)}{3 - 2} = 12 \text{ km/hr.}$

If a person with two different speeds U & V cover the same distance, then required distance

$$= \frac{U \times V}{U - V} \times \text{Difference between arrival time}$$

$$\text{Also, required distance} = \text{Total time taken} \times \frac{U \times V}{U + V}$$

Example 9 A boy walking at a speed of 10 km/h reaches his school 12 min late. Next time at a speed of 15 km/h reaches his school 7 min late. Find the distance of his school from his house?

Solution : Difference between the time = $12 - 7 = 5 \text{ min} = \frac{5}{60} = \frac{1}{12} \text{ hr}$

$$\text{Required distance} = \frac{15 \times 10}{15 - 10} \times \frac{1}{12}$$

$$= \frac{150}{5} \times \frac{1}{12} = 2.5 \text{ km}$$

A man leaves a point A at t_1 and reaches the point B at t_2 . Another man leaves the point B at t_3 and reaches the point A at t_4 , then they will meet at

$$t_1 + \frac{(t_2 - t_1)(t_4 - t_1)}{(t_2 - t_1) + (t_4 - t_3)}$$

Example 10 A bus leaves Ludhiana at 5 am and reaches Delhi at 12 noon. Another bus leaves Delhi at 8 am and reaches Ludhiana at 3 pm. At what time do the buses meet?

Solution : Converting all the times into 24 hour clock time, we get
 5 am = 500, 12 noon = 1200, 8 am = 800 and 3 pm = 1500

$$\text{Required time} = 500 + \frac{(1200 - 500)(1500 - 500)}{(1200 - 500) + (1500 - 800)}$$

$$= 500 + \frac{700 \times 1000}{700 + 700} = 1000 = 10 \text{ am.}$$

Relation between time taken with two different modes of transport : $t_{2x} + t_{2y} = 2(t_x + t_y)$

where,

t_x = time when mode of transport x is used single way.

t_y = time when mode of transport y is used single way.

t_{2x} = time when mode of transport x is used both ways.

t_{2y} = time when mode of transport y is used both ways.

Example 11 A man takes 6 hours 30 min. in going by a cycle and coming back by scooter. He would have lost 2 hours 10 min by going on cycle both ways. How long would it take him to go by scooter both ways?

Solution : Clearly, time taken by him to go by scooter both way =

$$6\text{h.}30\text{m} - 2\text{h.}10\text{m} = 4\text{h.}20\text{m} = 4\frac{1}{3} \text{ h}$$

Example 12 A man travels 120 km by ship, 450 km by rail and 60 km by horse taking altogether 13 hrs 30 min. The speed of the train is 3 times that of the horse and $1\frac{1}{2}$ times that of the ship. Find the speed of the train.

Solution : If the speed of the horse is x km/hr; that of the train is

$$3x \text{ and that of the ship is } \frac{3x}{1\frac{1}{2}} = 2x \text{ km/hr}$$

$$\therefore \frac{120}{2x} + \frac{450}{3x} + \frac{60}{x} = \frac{27}{2}$$

$$\therefore \frac{60}{x} + \frac{150}{x} + \frac{60}{x} = \frac{27}{2} \quad \therefore \quad \frac{270}{x} = \frac{27}{2}$$

$$\therefore x = 20 \quad \therefore \text{Speed of the train} = 60 \text{ km/hr.}$$

Chapter 15: Trains, Boats and Streams

A train is said to have crossed an object (stationary or moving) only when the last coach of the train crosses the said object completely. It implies that the total length of the train has crossed the total length of the object.

Time taken by a train to cross a pole/a standing man

$$= \frac{\text{Length of train}}{\text{Speed of train}}$$

Time taken by a train to cross platform/bridge etc. (i.e. a stationary object with some length)

$$= \frac{\text{length of train} + \text{length of platform/bridge etc.}}{\text{speed of train}}$$

When two trains with lengths L_1 and L_2 and with speeds S_1 and S_2 respectively, then

(a) When they are moving in the same direction, time taken by the faster train to cross the slower train

$$= \frac{L_1 + L_2}{\text{difference of their speeds}}$$

(b) When they are moving in the opposite direction, time taken by the trains to cross each other

$$= \frac{L_1 + L_2}{\text{sum of their speeds}}$$

Suppose two trains or two bodies are moving in the same direction at u km/hr and v km/hr respectively such that $u > v$, then

their relative speed = $(u - v)$ km/hr.

If their lengths be x km and y km respectively, then time taken by the faster train to cross the slower train (moving in the

same direction) = $\left(\frac{x+y}{u-v}\right)$ hrs.

Suppose two trains or two bodies are moving in opposite directions at u km/hr and v km/hr, then their relative speed = $(u + v)$ km/hr.

If their lengths be x km & y km, then :

time taken to cross each other = $\left(\frac{x+y}{u+v}\right)$ hrs.

If a man is running at a speed of u m/sec in the same direction in which a train of length L meters is running at a speed v m/sec, then $(v - u)$ m/sec is called the speed of the train relative to man. Then the time taken by the train to cross the man =

$$\frac{1}{v-u} \text{ seconds}$$

If a man is running at a speed of u m/sec in a direction opposite to that in which a train of length L meters is running with a speed v m/sec, then $(u + v)$ is called the speed of the train relative to man.

Then the time taken by the train to cross the man

$$= \frac{1}{v+u} \text{ seconds.}$$

If two trains start at the same time from two points A and B towards each other and after crossing, they take a and b hours in reaching B and A respectively. Then,

$$A's \text{ speed} : B's \text{ speed} = (\sqrt{b} : \sqrt{a})$$

Example 18 How long does a train 90 m long running at the rate of 54 km/h take to cross –

- a Mahatma Gandhi's statue ?
- a platform 120 m long ?
- another train 150m long, standing on another parallel track ?
- another train 160 m long running at 36 km/h in same direction ?
- another train 160 m long running at 36 km/h in opposite direction ?
- a man running at 6 km/h in same direction ?
- a man running at 6 km/h in opposite direction ?

Solution : (a) The statue is a stationary object, so time taken by train is same as time taken by train to cover a distance equal to its own length.

$$\text{Now, } 54 \text{ km/h} = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

$$\therefore \text{ Required time} = \frac{90}{15} = 6 \text{ sec.}$$

(b) The platform is stationary of length = 120 m.

Length to be covered

$$= \text{Length of the train} + \text{Length of the platform}$$

$$= 90 + 120 = 210 \text{ m}$$

$$\therefore \text{ Required time} = \frac{210}{15} = 14 \text{ sec.}$$

(c) Length to be covered

$$= \text{Length of the train} + \text{length of the other train}$$

$$= 90 + 150 = 240 \text{ m.}$$

$$\therefore \text{ Required time} = \frac{240}{15} = 16 \text{ sec.}$$

(d) Another train is moving in same direction.

Length to be covered

$$= \text{Length of the train} + \text{length of the other train}$$

$$= 90 + 160 = 250 \text{ m}$$

$$\text{Relative speed} = 54 - 36 = 18 \text{ kmph.}$$

$$\therefore \text{ Required time} = \frac{250}{18 \times \frac{5}{18}} = 50 \text{ sec.}$$

(e) Another train is moving in opposite direction.

Length to be covered

$$= \text{Length of the train} + \text{length of the other train}$$

$$= 90 + 160 = 250 \text{ m}$$

$$\text{Relative speed} = 54 + 36 = 90 \text{ kmph}$$

$$\therefore \text{ Required speed} = \frac{250}{\frac{5}{18} \times 90} = 10 \text{ sec.}$$

- (f) The man is moving in same direction,
so Length to be covered = Length of the train,
and relative speed = speed of train – speed of man

$$\therefore \text{Required time} = \frac{90}{(54-6) \times \frac{5}{18}} \\ = \frac{90}{40 \times 3} = \frac{27}{4} = 6\frac{3}{4} \text{ sec.}$$

- (g) The man is moving in opposite direction, so
Length to be covered = Length of the train, and
relative speed = speed of train + speed of man

$$\therefore \text{Required time} = \frac{90}{(54+6) \times \frac{5}{18}} = \frac{27}{5} = 5\frac{2}{5} \text{ sec.}$$

Example 19 Two trains of equal lengths are running on parallel tracks in the same direction at 46 km/h and 36 km/h, respectively. The faster train passes the slower train in 36 sec. What is the length of train?

Solution : Let the length of each train be x metres.

Then, the total distance covered = $(x+x) = 2x$ m

$$\text{Relative speed} = (46-36) = 10 \text{ km/h} = \frac{10 \times 5}{18} \text{ m/s}$$

$$\text{Now, } 36 = \frac{2x \times 18}{50} \text{ or } x = 50 \text{ m}$$

Example 20 A train 110 m in length travels at 60 km/h. How much time does the train take in passing a man walking at 6 km/h against the train?

Solution : Relative speeds of the train and the man

$$= (60+6) = 66 \text{ km/h} = \frac{66 \times 5}{18} \text{ m/s}$$

Distance = 110 m

Therefore, time taken in passing the men

$$= \frac{110 \times 18}{66 \times 5} = 6 \text{ s}$$

Example 21 Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

Solution : Relative speed of the trains

$$= (42+48) \text{ kmph} = 90 \text{ kmph}$$

$$= \left(90 \times \frac{5}{18} \right) \text{ m/sec} = 25 \text{ m/sec.}$$

Time taken by the trains to pass each other

= Time taken to cover $(137+163)$ m at 25 m/sec

$$= \left(\frac{300}{25} \right) \text{ sec} = 12 \text{ seconds.}$$

BOATS AND STREAM

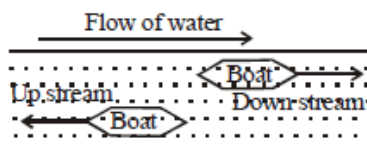
INTRODUCTION

Stream : It implies that the water in the river is moving or flowing.

Upstream : Going against the flow of the river.

Downstream : Going with the flow of the river.

Still water : It implies that the speed of water is zero (generally, in a lake).



Let the speed of a boat (or man) in still water be X m/sec and the speed of the stream (or current) be Y m/sec. Then,

Speed of boat with the stream (or downstream or D/S)
= $(X+Y)$ m/sec.

Speed of boat against the stream (or upstream or U/S)
= $(X-Y)$ m/sec.

Speed of boat in still water is

$$X = \frac{(X+Y) + (X-Y)}{2} = \frac{\text{Upstream} + \text{Downstream}}{2}$$

$$\text{Speed of the stream or current is } Y = \frac{(X+Y) - (X-Y)}{2} \\ = \frac{\text{Downstream} - \text{Upstream}}{2}$$

Example 22 A boat is rowed down a river 28 km in 4 hours and up a river 12 km in 6 hours. Find the speed of the boat and the river.

Solution : Downstream speed is $\frac{28}{4} = 7$ kmph

$$\text{Upstream speed is } \frac{12}{6} = 2 \text{ kmph}$$

$$\text{Speed of Boat} = \frac{1}{2} (\text{Downstream} + \text{Upstream Speed}) \\ = \frac{1}{2} [7+2] = 4.5 \text{ kmph}$$

$$\text{Speed of current} = \frac{1}{2} (\text{Downstream} - \text{Upstream speed}) \\ = \frac{1}{2} (7-2) = 2.5 \text{ kmph}$$

Example 23 P, Q, and R are the three towns on a river which flows uniformly. Q is equidistant from P and R. I row from P to Q and back in 10 hours and I can row from P to R in 4 hours. Compare the speed of my boat in still water with that of the river.

Solution : Let the speed of the boat be v_1 and the speed of the current be v_2 and d be the distance between the cities.

$$\text{Now, } \frac{d}{v_1 + v_2} = 4 \text{ and } \frac{d}{v_1 - v_2} = 6$$

$$\Rightarrow \frac{v_1 + v_2}{v_1 - v_2} = \frac{6}{4}$$

$$\text{or } \frac{2v_1}{2v_2} = \frac{10}{2} \quad \text{or } \frac{v_1}{v_2} = 5:1$$

$$\text{Required ratio} = (5+1):5 = 6:5$$

A man can row X km/h in still water. If in a stream which is flowing of Y km/h, it takes him Z hours to row to a place and

back, the distance between the two places is $\frac{Z(X^2 - Y^2)}{2X}$

Example 24 A man can row 6 km/h in still water. When the river is running at 1.2 km/h, it takes him 1 hour to row to a place and back. How far is the place?

Solution : Man's rate downstream = $(6 + 1.2) = 7.2$ km/h.

Man's rate upstream = $(6 - 1.2)$ km/h = 4.8 km/h.

Let the required distance be x km.

$$\text{Then } \frac{x}{7.2} + \frac{x}{4.8} = 1 \text{ or } 4.8x + 7.2x = 7.2 \times 4.8$$

$$\Rightarrow x = \frac{7.2 \times 4.8}{12} = 2.88 \text{ km}$$

Quicker Method

$$\begin{aligned} \text{Required distance} &= \frac{1 \times (6^2 - (1.2)^2)}{2 \times 6} \\ &= \frac{36 - 1.44}{12} = \frac{34.56}{12} = 2.88 \text{ km} \end{aligned}$$

A man rows a certain distance downstream in X hours and returns the same distance in Y hours. If the stream flows at the rate of Z km/h, then the speed of the man in still water is given by

$$\frac{Z(X+Y)}{Y-X} \text{ km/hr}$$

And if speed of man in still water is Z km/h then the speed of stream is given by

$$\frac{Z(Y-X)}{X+Y} \text{ km/hr}$$

Example 25 Vikas can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of 3 km/h, find the speed of Vikas in still water.

Solution : Quicker Method

$$\text{Vikas's speed in still water} = \frac{3(9+6)}{9-6} = 15 \text{ km/h}$$

If a man capable of rowing at the speed u of m/sec in still water, rows the same distance up and down a stream flowing at a rate of v m/sec, then his average speed through the journey is

$$= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}} = \frac{(u-v)(u+v)}{u}$$

Example 26 Two ferries start at the same time from opposite sides of a river, travelling across the water on routes at right angles to the shores. Each boat travels at a constant speed though

their speeds are different. They pass each other at a point 720m from the nearer shore. Both boats remain at their sides for 10 minutes before starting back. On the return trip they meet at 400m from the other shore. Find the width of the river.

Solution : Let the width of the river be x and a, b be the speeds of the ferries.

$$\frac{720}{a} = \frac{(x-720)}{b} \quad \dots (i)$$

$$\frac{(x-720)}{a} + 10 + \frac{400}{a} = \frac{720}{b} + 10 + \frac{(x-400)}{b} \quad \dots (ii)$$

(Time for ferry 1 to reach other shore + 10 minute wait + time to cover 400m)

= Time for ferry 2 to cover 720m to other shore + 10 minute wait + Time to cover $(x-400)$ m)

$$\text{Using (i), we get } \frac{a}{b} = \frac{720}{(x-720)}$$

$$\text{Using (ii), } \frac{(x-320)}{a} = \frac{(x+320)}{a} \Rightarrow \frac{a}{b} = \frac{(x-320)}{(x+320)}$$

On, solving we get, $x = 1760$ m

Example 27 A man rows 27km with the stream and 15km against the stream taking 4 hours each time. Find this rate per hour in still water and the rate at which the stream flows.

$$\text{Solution : Speed with the stream} = \frac{27}{4} = 6\frac{3}{4} \text{ kmph}$$

$$\therefore \text{Speed against the stream} = \frac{15}{4} = 3\frac{3}{4} \text{ kmph.}$$

$$\therefore \text{Speed of the man in still water}$$

$$= \frac{1}{2} \left(6\frac{3}{4} + 3\frac{3}{4} \right) = 5\frac{1}{4} \text{ kmph}$$

$$\therefore \text{Speed of the stream} = \frac{1}{2} \left(6\frac{3}{4} - 3\frac{3}{4} \right) = 1.5 \text{ kmph}$$

Example 28 On a river, B is between A and C and is also equidistant from A and C . A boat goes from A to B and back in 5 hours 15 minutes and from A to C and back in 7 hours. How long will it take to go from C to A if the river flows from A to C ?

Solution : If the speed in still water is x kmph and speed of the river is y kmph, speed down the river = $x+y$ and speed up the river $x-y$.

$$\therefore \frac{d}{x+y} + \frac{d}{x-y} = 5\frac{1}{4} \quad \dots (1)$$

$$\frac{2d}{x+y} = 7 \quad \dots (2)$$

$$\text{Multiplying (1) by 2, we get } \frac{2d}{x+y} + \frac{2d}{x-y} = 10\frac{1}{2}$$

$$\Rightarrow 7 + \frac{2d}{x-y} = \frac{21}{2} \quad \left[\because \frac{2d}{x+y} = 7 \right]$$

$$\Rightarrow \frac{2d}{x-y} = 3\frac{1}{2} \text{ hours} = \text{Time taken to travel from } C \text{ to } A.$$

Chapter 16: Permutation and Combination

PERMUTATION

Factorial : The important mathematical term "Factorial" has extensively used in this chapter.

The product of first n consecutive natural numbers is defined as factorial of n . It is denoted by $n!$ or $\underline{n}$. Therefore,

$$n! = 1 \times 2 \times 3 \times \dots \times (n-1) \times n$$

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$$\frac{n!}{r!} \neq \left(\frac{n}{r}\right)!$$

$$0! = 1$$

The factorials of fractions and negative integers are not defined.

Example 1 Prove that $n! + 1$ is not divisible by any natural number between 2 and ' n '.

Solution : Since $n! = 1 \cdot 2 \cdot 3 \cdot 4 \dots (n-1) \cdot n$

Therefore $n!$ is divisible by any number from 2 to n .

Consequently $n! + 1$, when divided by any number between 2 and n leaves 1 as remainder.

Hence, $n! + 1$ is not divisible by any number between 2 and n .

FUNDAMENTAL PRINCIPLES OF COUNTING

Principle of Addition : If an event can occur in m ways and another event can occur in n ways independent of the first event, then either of the two events can occur in $(m + n)$ ways.

Principle of Multiplication : If an operation can be performed in m ways and after it has been performed in any one of these ways, a second operation can be performed in n ways, then the two operations in succession can be performed in $(m \times n)$ ways.

Example 2 In a class there are 10 boys and 8 girls. The class teacher wants to select a student for monitor of the class. In how many ways the class teacher can make this selection ?

Solution : The teacher can select a student for monitor in two exclusive ways

- (i) Select a boy among 10 boys, which can be done in 10 ways OR

- (ii) Select a girl among 8 girls, which can be done in 8 ways.
Hence, by the fundamental principle of addition, either a boy or a girl can be selected in $10 + 8 = 18$ ways.

Example 3 In a class there are 10 boys and 8 girls. The teacher wants to select a boy and a girl to represent the class in a function. In how many ways can the teacher make this selection ?

Solution : The teacher has to perform two jobs :

- (i) To select a boy among 10 boys, which can be done in 10 ways.
(ii) To select a girl, among 8 girls, which can be done in 8 ways.

Hence, the required number of ways $= 10 \times 8 = 80$.

Example 4 There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 5 choices each ?

Solution : Each of the first three questions can be answered in 4 ways and each of the next three questions can be answered in 5 different ways.

Hence, the required number of different sequences of answers $= 4 \times 4 \times 4 \times 5 \times 5 \times 5 = 8000$.

Example 5 Five persons entered a lift cabin on the ground floor of an 8-floor house. Suppose that each of them can leave the cabin independently at any floor beginning with the first. What is the total number of ways in which each of the five persons can leave the cabin at any of the 7 floors ?

Solution : Any one of the 5 persons can leave the cabin in 7 ways independent of other.

Hence the required number of ways $= 7 \times 7 \times 7 \times 7 \times 7 = 7^5$.

Method of Sampling :

Sampling process can be divided into following forms :

1. The order is IMPORTANT and the repetition is ALLOWED, each sample is then a SEQUENCE.
2. The order is IMPORTANT and the repetition is NOT ALLOWED, each sample is then a PERMUTATION.

- The order is NOT IMPORTANT and repetition is ALLOWED, each sample is then a MULTISSET.
- The order is NOT IMPORTANT and repetition is NOT ALLOWED, each sample is then a COMBINATION.

PERMUTATION

Each of the arrangements, which can be made by taking, some or all of a number of things is called a PERMUTATION.

For Example : Formation of numbers, word formation, sitting arrangement in a row.

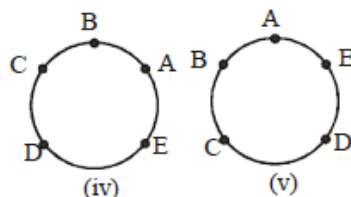
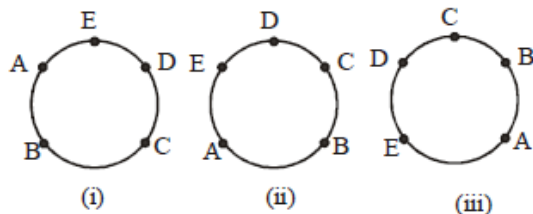
The number of permutations of n things taken r at a time is denoted by ${}^nP_r$. It is defined as, ${}^nP_r = \frac{n!}{(n-r)!}$.

$${}^nP_n = n!$$

Circular Permutations:

- Arrangements round a circular table :

Consider five persons A, B, C, D and E to be seated on the circumference of a circular table in order (which has no head). Now, shifting A, B, C, D and E one position in anticlockwise direction we will get arrangements as follows:



we see that arrangements in all figures are same.

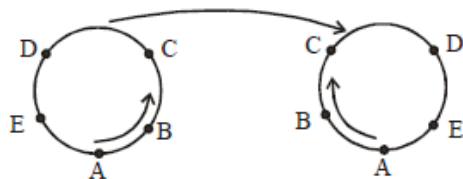
∴ The number of circular permutations of n different things

taken all at a time is $\frac{{}^nP_n}{n} = (n-1)!$, if clockwise and anticlockwise orders are taken as different.

- Arrangements of beads or flowers (all different) around a circular necklace or garland:

Consider five beads A, B, C, D and E in a necklace or five flowers A, B, C, D , and E in a garland etc. If the necklace or garland on the left is turned over we obtain the arrangement on the right, i.e., anticlockwise and clockwise order of arrangements are not different.

Thus the number of circular permutations of n different things taken.



All at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken to be same.

Example 6 The number of ways in which 10 persons can sit round a circular table so that none of them has the same neighbours in any two arrangements.

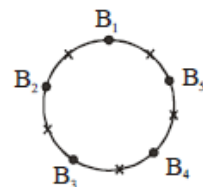
Solution :

10 persons can sit round a circular table in $9!$ ways. But here clockwise and anticlockwise orders will give the same neighbours. Hence the required number of ways = $\frac{1}{2}9!$

Example 7 In how many different ways can five boys and five girls form a circle such that the boys and girls are alternate ?

Solution : After fixing up one boy on the table the remaining can be arranged in $4!$ ways.

There will be 5 places, one



place each between two boys

which can be filled by 5 girls in $5!$ ways.

Hence by the principle of multiplication, the required number of ways = $4! \times 5! = 2880$.

Conditional Permutations

- Number of permutations of n things taking r at a time, in which a particular thing always occurs = $r \cdot {}^{n-1}P_{r-1}$.

Distinguishable Permutations

Suppose a set of n objects has n_1 of one kind of object, n_2 of a second kind, n_3 of a third kind, and so on, with $n = n_1 + n_2 + n_3 + \dots + n_k$. Then the number of distinguishable permutations of the n objects is

$$\frac{n!}{n_1! n_2! n_3! \dots n_k!}$$

COMBINATION

Each of the different selections that can be made with a given number of objects taken some or all of them at a time is called a Combination.

The number of combinations of n dissimilar things taken r at a time is denoted by ${}^nC_r$ or $C(n, r)$. It is defined as,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Example 8 In how many ways a hockey team of eleven can be elected from 16 players ?

Solution : Total number of ways = ${}^{16}C_{11} = \frac{16!}{11! \times 5!}$

$$= \frac{16 \times 15 \times 14 \times 13 \times 12}{5 \times 4 \times 3 \times 2 \times 1} = 4368.$$

REMEMBER

$${}^nC_0 = 1, {}^nC_n = 1; {}^nP_r = r! {}^nC_r$$

$${}^nC_r = {}^nC_{n-r}$$

$${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$$

$${}^nC_x = {}^nC_y \Rightarrow x = y = n$$

$${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$$

$${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$$

$${}^nC_r = \frac{1}{r} (n-r+1) {}^nC_{r-1}$$

$${}^nC_1 = {}^nC_{n-1} = n$$

Conditional Combinations

Number of combinations of n distinct things taking r ($\leq n$) at a time, when k ($0 \leq k \leq r$) particular objects always occur

$$= {}^{n-k}C_{r-k}$$

Number of combinations of n distinct objects taking r ($\leq n$) at a time, when k ($0 \leq k \leq r$) particular objects never occur

$$= {}^{n-k}C_r$$

Number of selections of r things from n things when p particular things are not together in any selection

$$= {}^nC_r - {}^{n-p}C_{r-p}$$

Number of selection of r consecutive things out of n things in a row

$$= n - r + 1$$

Number of selection of r consecutive things out of n things along a circle

$$= \begin{cases} n, & \text{when } r < n \\ 1, & \text{when } r = n \end{cases}$$

The number of Combinations of n different things taking some or all at a time

$$= {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1$$

Example 9 In a class of 25 students, find the total number of ways to select two representative,

- if a particular person will never be selected.
- if a particular person is always there.

Solution :

- Total students (n) = 25
A particular students will not be selected (p) = 1,
So total number of ways = ${}^{25-1}C_2 = {}^{24}C_2 = 276$.

- Using ${}^{n-p}C_{r-p}$ number of ways
 $= {}^{25-1}C_{2-1} = {}^{24}C_1 = 24$.

If a person is always there then we have to select only 1 from the remaining $25 - 1 = 24$

Example 10 There are 10 lamps in a hall. Each of them can be switched on independently. Find the number of ways in which the hall can be illuminated.

Solution :

Since each bulb has two choices, either switched on or off, therefore required number = $2^{10} - 1 = 1023$.

The number of ways of dividing ' $m + n$ ' things into two groups containing ' m ' and ' n ' things respectively

$$= {}^{m+n}C_m \cdot {}^nC_n = \frac{(m+n)!}{m!n!}$$

The number of ways of dividing ' $m + n + p$ ' things into three groups containing ' m ', ' n ' and ' p ' things respectively

$$= {}^{m+n+p}C_m \cdot {}^{n+p}C_p = \frac{(m+n+p)!}{m!n!p!}$$

- If $m = n = p$ i.e. $3m$ things are divided into three equal groups then the number of combinations is

$$\frac{(3m)!}{m!m!m!3!} = \frac{(3m)!}{(m!)^3 3!}$$

- But if ' $3m$ ' things are to be divided among three persons, then the number of divisions is

$$\frac{(3m)!}{(m!)^3}$$

If mn distinct objects are to be divided into m groups. Then, the number of combination is

$$\frac{(mn)!}{m! (n!)^m},$$

when the order of groups is not important and

$$\frac{(mn)!}{(n!)^m}, \text{ when the order of groups is important}$$

Example 11 Find the number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card.

Solution :

Here we have to divide 52 cards into 4 sets, three of them having 17 cards each and the fourth one having just one card. First we divide 52 cards into two groups of 1 card and 51 cards. this can be done in

$$\frac{52!}{1!51!} \text{ ways.}$$

Now every group of 51 cards can be divided into 3 groups of 17 each in

$$\frac{51!}{(17!)^3 3!} \text{ ways.}$$

Hence the required number of ways

$$= \frac{52!}{1!51!} \cdot \frac{51!}{(17!)^3 3!} = \frac{52!}{(17!)^3 3!}$$

NUMBER OF RECTANGLES AND SQUARES

- (a) Number of rectangles of any size in a square of size $n \times n$ is

$$\sum_{r=1}^n r^3$$

and number of squares of any size is $\sum_{r=1}^n r^2$.

- (b) Number of rectangles of any size in a rectangle size $n \times p$ ($n < p$) is $\frac{np}{4}(n+1)(p+1)$ and number of squares of any size is

$$\sum_{r=1}^n (n+1-r)(p+1-r).$$

Example 12 The number of squares that can be formed on a chessboard.

Solution :

A chessboard is made up of 9 equispaced horizontal and vertical line. To make a 1×1 square, we must choose two consecutive horizontal and vertical lines from among these. This can be done in $8 \times 8 = 8^2$ ways. A 2×2 square needs three consecutive horizontal and vertical lines, and we can do this in $7 \times 7 = 7^2$ ways. Continuing in this manner, the total number of square is

$$8^2 + 7^2 + 6^2 + \dots + 2^2 + 1^2 = \frac{8(8+1)[(2 \times 8) + 1]}{6} = 204.$$

FINDING THE RANK OF A WORD

We can find the rank of a word out of all the words with or without meaning formed by arranging all the letters of a given word in all possible ways when these words are listed as in a dictionary. You can easily understand the method to find the above mentioned rank by the following illustrations.

Example : If the letters of the word RACHIT are arranged in all possible ways and these words (with or without meaning) are written as in a dictionary, then find the rank of this word RACHIT.

Solution :

The order of the alphabet of RACHIT is A, C, H, I, R, T.

The number of words beginning with A (i.e. the number of words in which A comes at first place) is ${}^5P_5 = 5!$.

Similarly, number of words beginning with C is $5!$, beginning with H is $5!$ and beginning with I is also $5!$.

So before R, four letters A, C, H, I can occur in $4 \times (5!) = 480$ ways.

Now the word RACHIT happens to be the first word beginning with R. Therefore the rank of this word RACHIT = $480 + 1 = 481$.

Example : The letters of the word MODESTY are written in all possible orders and these words (with or without meaning) are listed as in a dictionary then find the rank of the word MODESTY.

Solution :

The order of the alphabet of MODESTY is D, E, M, O, S, T, Y.

Number of words beginning with D is ${}^6P_6 = 6!$

Number of words beginning with E is ${}^6P_6 = 6!$

Number of words beginning with MD is ${}^5P_5 = 5!$

Number of words beginning with ME is ${}^5P_5 = 5!$

Now the first word start with MO is MODESTY.

Hence rank of the word MODESTY

$$= 6! + 6! + 5! + 5! + 5! + 1$$

$$= 720 + 720 + 120 + 120 + 1$$

$$= 1681.$$

Chapter 17: Probability

Probability is the section of mathematics, that deals with the results of random events. The word probability means chance or possibility of an outcome. It explains the possibility of a particular event occurring. We often use sentences like - ‘It will probably rain today, ‘he will probably pass the test’, ‘there is very less probability of getting a storm tonight’, and ‘most probably the price of onion will go high again. In all these sentences, we replace words like chance, doubt, maybe, likely, etc., with the word probability. Probability is basically the prediction of an event that is either based on the study of previous records or the number and type of possible outcomes.

INTRODUCTION

Random Experiment : It is an experiment which if conducted repeatedly under homogeneous condition does not give the same result.

The total number of possible outcomes of an experiment in any trial is known as the **exhaustive number** of events.

For example

- (i) In throwing a die, the exhaustive number of cases is 6 since any one of the six faces marked with 1, 2, 3, 4, 5, 6 may come uppermost.
- (ii) In tossing a coin, the exhaustive number of cases is 2, since either head or tail may turn over.
- (iii) If a pair of dice is thrown, then the exhaustive number of cases is $6 \times 6 = 36$

- (iv) In drawing four cards from a well-shuffled pack of cards, the exhaustive number of cases is ${}^{52}C_4$.

Events are said to be **mutually exclusive** if no two or more of them can occur simultaneously in the same trial.

For example,

- (i) In tossing of a coin the events head (H) and tail (T) are mutually exclusive.
- (ii) In throwing of a die all the six faces are mutually exclusive.
- (iii) In throwing of two dice, the events of the face marked 5 appearing on one die and face 5 (or other) appearing on the other are not mutually exclusive.

Outcomes of a trial are **equally likely** if there is no reason for an event to occur in preference to any other event or if the chances of their happening are equal.

For example,

- In throwing of an unbiased die, all the six faces are equally likely to occur.
- In drawing a card from a well-shuffled pack of 52 cards, there are 52 equally likely possible outcomes.

The **favourable** cases to an event are the outcomes, which entail the happening of an event.

For example,

- In the tossing of a die, the number of cases which are favourable to the "appearance of a multiple of 3" is 2, viz. 3 and 6.
- In drawing two cards from a pack, the number of cases favourable to "drawing 2 aces" is ${}^4C_2$.
- In throwing of two dice, the number of cases favourable to "getting 8 as the sum" is 5, i.e. (2, 6), (6, 2), (4, 4), (3, 5), (5, 3).

Events are said to be **independent** if the happening (or non-happening) of one event is not affected by the happening or non-happening of others.

CLASSICAL DEFINITION OF PROBABILITY

If there are n -mutually exclusive, exhaustive and equally likely outcomes to a random experiment and ' m ' of them are favourable to an event A , then the probability of happening of A is denoted

by $P(A)$ and is defined by $P(A) = \frac{m}{n}$.

$$P(A) = \frac{\text{No. of elementary events favourable to } A}{\text{Total no. of equally likely elementary events}}$$

Obviously, $0 \leq m \leq n$, therefore $0 \leq \frac{m}{n} \leq 1$ so that

$$0 \leq P(A) \leq 1.$$

$P(A)$ can never be negative.

Since, the number of cases in which the event A will not happen is ' $n - m$ ', then the probability $P(\bar{A})$ of not happening of A is given by

$$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$$\Rightarrow P(A) + P(\bar{A}) = 1$$

The **ODDS IN FAVOUR** of occurrence of A are given by

$$m : (n-m) \text{ or } P(A) : P(\bar{A})$$

The **ODDS AGAINST** the occurrence of A are given by

$$(n-m) : m \text{ or } P(\bar{A}) : P(A).$$

Example 13 Two dice are thrown simultaneously. What is the probability of obtaining a total score of seven?

Solution : When two are thrown then there are 6×6 exhaustive cases $\therefore n = 36$.

Let A denote the event "total score of 7" when 2 dice are thrown then $A = [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)]$.

Thus there are 6 favourable cases.

$$\therefore m = 6 \quad \text{By definition } P(A) = \frac{m}{n}$$

$$\therefore P(A) = \frac{6}{36} = \frac{1}{6}.$$

Example 14 A bag contains 5 green and 7 red balls. Two balls are drawn. Find the probability that one is green and the other is red.

Solution : There are $5 + 7 = 12$ balls in the bag and out of these two balls can be drawn in ${}^{12}C_2$ ways. There are 5 green balls, therefore, one green ball can be drawn in ${}^5C_1$ ways; similarly, one red ball can be drawn in ${}^7C_1$ ways so that the number of ways in which we can draw one green ball and the other red is ${}^5C_1 \times {}^7C_1$. Hence, P (one green and the other red)

$$= \frac{{}^5C_1 \times {}^7C_1}{{}^{12}C_2} = \frac{5}{1} \times \frac{7}{1} \times \frac{1.2}{12.11} = \frac{35}{66}$$

Example 15 The letters of the word SOCIETY are placed at random in a row. What is the probability that the three vowels come together?

Solution : The word 'SOCIETY' contains seven distinct letters and they can be arranged at random in a row in ${}^7P_7$ ways, i.e., in $7! = 5040$ ways.

Let us now consider those arrangements in which all the three vowels come together. So in this case we have to arrange four letters, S, C, T, Y and a pack of three vowels in a row which can be done in ${}^5P_5$ i.e., $5! = 120$ ways.

Also, the three vowels in their pack can be arranged in ${}^3P_3$ i.e. $3! = 6$ ways.

Hence, the number of arrangements in which the three vowels come together is $120 \times 6 = 720$

$\therefore$ The probability that the vowels come together

$$= \frac{720}{5040} = \frac{1}{7}$$

Example 16 There are three events E_1, E_2 and E_3 , one of which must, and only one can happen. The odds are 7 to 4 against E_1 and 5 to 3 against E_2 . Find the odds against E_3 .

Solution : Since, one and only one of the three events E_1, E_2 and E_3 can happen, therefore $P(E_1) + P(E_2) + P(E_3) = 1 \dots (i)$

$\therefore$ Odds against E_1 are 7 : 4

$$\Rightarrow P(E_1) = \frac{4}{4+7} = \frac{4}{11} \dots (ii)$$

$\therefore$ Odds against E_2 are 5 : 3

$$\Rightarrow P(E_2) = \frac{3}{3+5} = \frac{3}{8} \dots (iii)$$

From (1), (2) and (3), we have, $\frac{4}{11} + \frac{3}{8} + P(E_3) = 1$.

$$\text{i.e., } P(E_3) = 1 - \frac{4}{11} - \frac{3}{8} = \frac{88 - 32 - 33}{88} = \frac{23}{88} = \frac{23}{23 + 65}$$

Hence odds against E_3 is 65 : 23.

ALGEBRA OF EVENTS

Let A and B be two events related to a random experiment. We define

- (i) The event " A or B " denoted by " $A \cup B$ ", which occurs when A or B or both occur. Thus,

$P(A \cup B)$ = Probability that at least one of the events occur

- (ii) The event A and B , denoted by $A \cap B$, which occurs when A and B both occur. Thus,

$P(A \cap B)$ = Probability of simultaneous occurrence of A and B .

- (iii) The event " $\text{Not } A$ " denoted by $\bar{A}$, which occurs when and only when A does not occur. Thus

$P(\bar{A})$ = Probability of non-occurrence of the event A .

- (iv) $\bar{A} \cap \bar{B}$ denotes the " $\text{non-occurrence of both } A \text{ and } B$ ".

- (v) " $A \subset B$ " denotes the " $\text{occurrence of } A \text{ implies the occurrence of } B$ ".

For example :

Consider a single throw of die and following two events

A = the number is even = $\{2, 4, 6\}$

B = the number is a multiple of 3 = $\{3, 6\}$

$$\text{Then } P(A \cup B) = \frac{4}{6} = \frac{2}{3}, \quad P(A \cap B) = \frac{1}{6}$$

$$P(\bar{A}) = \frac{1}{2}, \quad P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - \frac{2}{3} = \frac{1}{3}.$$

ADDITION THEOREM ON PROBABILITY

1. Addition Theorem : If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. Addition Theorem for three Events : If A, B, C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

3. If A and B are two mutually exclusive events and the probability of their occurrence are $P(A)$ and $P(B)$ respectively, then probability of either A or B occurring is given by

$$P(A \text{ or } B) = P(A) + P(B) \\ \Rightarrow P(A + B) = P(A) + P(B)$$

Example 17 A and B are two events odds against A are 2 to 1. Odds in favour of $A \cup B$ are 3 to 1. If $x \leq P(B) \leq y$ find then, the ordered pair (x, y) .

Solution :

$$P(A) = \frac{1}{3}; \quad P(A \cup B) = \frac{3}{4}$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{3}{4}$$

$$\Rightarrow \frac{1}{3} + P(B) - P(A \cap B) = \frac{3}{4}$$

$$\Rightarrow P(B) = \frac{5}{12} + P(A \cap B) \geq \frac{5}{12}$$

$$\text{Also, } P(B) = \frac{5}{12} + P(A \cap B) \leq \frac{5}{12} + \frac{1}{3} = \frac{3}{4}$$

$$\left[\because P(A \cap B) \leq P(A) = \frac{1}{3} \right]$$

$$\text{Hence, } (x, y) \text{ is } \left(\frac{5}{12}, \frac{3}{4} \right).$$

Example 18 Two cards are drawn from a pack of 52 cards. Find the probability that either both are red or both are kings.

Solution : 2 cards can be drawn from the pack in ${}^{52}C_2$ ways. Let A be the event " Two cards are red " and B be the event " $\text{Two cards drawn are kings}$ ".

The required probability is $P(A \cup B)$.

From addition theorem, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B). \quad \dots(i)$$

Now, $P(A)$ = Probability of drawing two red cards

$$= \frac{{}^{26}C_2}{{}^{52}C_2} \quad [\because \text{There are total 26 red cards}]$$

$P(B)$ = Probability of drawing two king cards

$$= \frac{{}^4C_2}{{}^{52}C_2} \quad [\because \text{There are 4 king cards}]$$

$P(A \cap B)$ = Probability of drawing 2 red king cards

$$= \frac{{}^2C_2}{{}^{52}C_2} \quad [\because \text{There are just 2 red kings}]$$

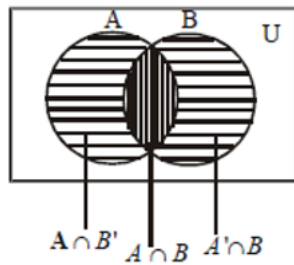
Substituting the values in (i), we get

$$P(A \cup B) = \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^4C_2}{{}^{52}C_2} - \frac{{}^2C_2}{{}^{52}C_2} = \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} \\ = \frac{55}{221}.$$

Example 19 If A and B are two events, the probability that at most one of these events occurs is :

- $P(A') + P(B') - P(A' \cap B')$
- $P(A') + P(B') + P(A \cup B) - 1$
- $P(A \cap B') + P(A' \cap B) + P(A' \cap B')$
- All above are correct.

Solution : (d)



At most one of two events occurs if the event $A \cup B'$ occurs.

$$P(A \cup B') = 1 - P(A \cap B)$$

Now,
$$P(A \cup B') = P(A) + P(B') - P(A' \cap B')$$

$$= P(A) + P(B') - [1 - P(A \cup B)]$$

$$= P(A) + P(B') + P(A \cup B) - 1.$$

Finally, since

$$\begin{aligned} P(A \cup B') &= P[(A')' \cap B'] + P[A' \cap (B')'] + P(A' \cap B') \\ &= P(A \cap B') + P(A' \cap B) + P(A' \cap B') \end{aligned}$$

$$[\because P(A \cup B) = P(A' \cap B) + P(A \cap B') + P(A \cap B)]$$

[See the Venn diagram].

Logical Reasoning

Chapter 01: Coding and Decoding

CODING

The word 'coding' stands for converting a word from English language into a certain pattern or expression.

Therefore, code is a sequence of letters/numbers, which is used in place of the original word/series of numbers that is coded.

Coding can be done for a group of letters (a word), a series of numbers or an alphanumeric series (i.e., a series having both alphabets as well as numerals).

There are 4 types of coding methods :

I Simple Arrangement.

II Direct Substitution.

III Pattern Substitution.

IV Alphanumeric coding.

I Simple Arrangement Method : This is the most common & the simplest kind of coding. These codes are generally obtained by simply re-aligning the given alphabets in a word.

Example 1 In a code language, if TRAINS is coded as RTIASN, how will FLOWER be coded in the same language ?

- (a) LFLOWER (b) LFWORE
(c) WORELF (d) ERFLOW

Solutions. (b) TRAINS $\longrightarrow$ RTIASN

In the above code, we can clearly observe that the code is obtained simply by interchanging the positions of 2 consecutive alphabets i.e., TR becomes RT, AI becomes IA and NS becomes SN similarly,

FLOWER will be coded as (FL becomes LF, OW becomes WO and ER becomes RE) LFWORE.

FLOWER $\longrightarrow$ LFWORE

Therefore, correct answer is option (b).

Example 2 If the word 'MECHANIC' is coded as 'EMHCNACT' in a certain code language, then how will 'WESTSIDE' be coded in the same language ?

- (a) EWTSISED (b) EWTSSIED
(c) EWTSISDE (d) WETSSIED

Solutions. (a) MECHANIC $\longrightarrow$ EMHCNACT

Similar to example 1, in the word MECHANIC also we have done the same thing. ME becomes EM, CH becomes HC, AN becomes NA & IC becomes CI.

Similarly, in the word 'WESTSIDE', WE becomes EW, ST becomes TS, SI becomes IS and DE becomes ED.

So, WESTSIDE $\longrightarrow$ EWTSISED

Hence, correct answer is option (a).

Example 3 In a certain code language, the word 'PARTNER' is coded as 'TRAPREN', how will 'FOUNDER' be coded in the same language.

- (a) NUOFDER (b) NUOFRED
(c) FOUNRED (d) OFNUEDR

Solutions. (b) When we divide letters of the word 'PARTNER' in two group.

First group of first 4 letters and second group of last 3 letters, i.e., PART and NER and then reverse the order of the letters in these two groups.

PART $\longrightarrow$ TRAP
NER $\longrightarrow$ REN $\Rightarrow$ TRAPREN

Similarly, we divide the letters of the word 'FOUNDER' in two group.

First group with first 4 letters (FOUN) and second group with last 3 letters (DER) & then reverse the order of the letters in the 2 groups.

FOUN $\longrightarrow$ NUOF DER $\longrightarrow$ RED

$\therefore$ the code for FOUNDER is NUOFRED

FOUN DER
 X X
NUOF RED

Therefore, correct answer is option (b).

QUICK TIPS

Notes for simple arrangement :

- (i) Number of characters (letters / numbers / symbols) in the code should be same as that of the original word, otherwise coding is not possible.
- (ii) Pay attention to the alignment of the word, i.e., if the letter/number has changed its position from first to last & vice versa, a swap coding is possible.

For e.g. : in e.g. 3, first group had 4 letters — 'FOUN' & there was a swap between letters at first & last position.

FOUN
 X F
N] Swap coding

II. Direct Substitution Method : When the characters, *i.e.*, letters of a word or numerals of a series are substituted by a coded character, *i.e.*, an alphabet, a numeral or a symbol & are placed in the coded word at similar positions as in the given / original word, it is known as direct substitution method.

These codes (substitutions) may either be in a direct fashion or in a jumbled fashion in order to make the questions tricky.

Example 4 In a code language, if SUGAR is coded as ZNMBD and TEA is coded as FLD, how would you code GRATE in the same code language.

- (a) BNDFL (b) MBDFL
(c) LDZMN (d) FLDZB

Solutions. (b) Original word : SUGAR TEA
↓ ↓ ↓ ↓ ↓ ↓
Coded word : ZNMBD FLD

First we write the original words SUGAR & TEA and its corresponding alignment to these, we write down their codes respectively.

Therefore, we see that Z is coded for S, N for U, M for G, D for A and B for R, in the word SUGAR. While in the word TEA, F is substituted for T, L for E & D for A.

This implies that this code is in direct fashion as in both the words 'D' is coded for A.

Therefore, code for GRATE will be MBDFL.

GRATE
↓ ↓ ↓ ↓ ↓
MBDFL

So, correct answer option is (b).

Example 5 If in a certain code language, TWENTY is coded as 863985 and ELEVEN is coded as 323039, how will TWELVE be coded?

- (a) 863903 (b) 86365
(c) 863203 (d) 683583

Solutions. (c) Original words : TWENTY ELEVEN
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
Coded words : 8 6 3 9 8 5 3 2 3 0 3 9

Therefore, in the word TWELVE, 8 will be the code for T, 6 will be the code for W, 3 for E, 2 for L, and 0 for V.

So, TWELVE
↓ ↓ ↓ ↓ ↓
8 6 3 2 0 3

Hence, answer option (b) is the correct answer.

Shortcut : Code for T is 8, so the code cannot start with '6'. So option (d) is ruled out, option (b) has only 5 numbers while the word has 6 alphabets, therefore, option (b) is also ruled out. Code for L is '2', so option (c) is the correct answer.

QUICK TIPS

Notes for direct substitution :

- If there are 2 words in the question for which codes are given and these 2 words have 1 or more same alphabets, then the codes for these alphabets will be the same as well.
- In case of confusion, note (a), *i.e.*, same codes in both the words for same alphabet, will help us identify that the question belongs to the category of direct substitution.

III. Pattern Substitution Method : This method involves the use of the alphabet series (A B C D E F X Y Z).

A certain word will be coded as certain other letters from the alphabet series following a certain pattern. In substitution coding some particular objects are assigned as code names and then a question is asked that is to be answered in the code language. It is further explained by examples.

Example 6 In a certain code language, the word 'RECTANGLE' is coded as TGEVCPING, then how is the word 'RHOMBUS' coded?

- (a) TJOQDWV (b) UVWTJQN
(c) TJQODWU (d) JTQOEWN

Solutions. (c) Each letter of the word RECTANGLE is moved two steps forward to obtain the corresponding letters of the code, *i.e.*,

R E C T A N G L E
↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2
T G E V C P I N G

Similarly, we have :

R H O M B U S
↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2
T J Q O D W U

So, the desired code is 'TJQODWU'. So, option (c) is the correct answer.

Shortcut : After we obtain the pattern of substitution, *i.e.*, after the step.

R E C T A N G L E
↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2 ↓ +2
T G E V C P I N G &

Since we know that the word to be coded 'RHOMBUS' starts with the same alphabets as 'RECTANGLE', *i.e.*, both start with R therefore the code for both will be the same *i.e.*, T. So, options (b) & (d) are ruled out.

Since in options (a) & (c), the starting alphabet is the same but the last alphabet is different, we directly check for the last alphabet of the word 'RHOMBUS', *i.e.*, S.

S $\xrightarrow{+2}$ U. Hence option (c) is the correct answer.

Example 7 If wall is called window, window is called door, door is called floor, floor is called roof and roof is called ventilator, what will a person stand on?

- (a) door (b) ventilator
(c) Roof (d) Floor

Sol. (c) A person stands on the floor and in the given code language, floor is called roof.
Hence, roof will be the correct answer.

Example 8 In a certain language, the word 'REJECT' is written as SGMIHJ. How will the word 'ACCEPT' be written?

- (a) BEFIUV (b) BEFMUZ
(c) BEFIUZ (d) BEFIUZ

Solutions. (d) Clearly, the pattern followed by the code is moving up in an increasing order.

R	E	J	E	C	T
↓ +1	↓ +2	↓ +3	↓ +4	↓ +5	↓ +6
S	G	M	I	H	J

Similarly, code for the word 'ACCEPT' will be

A	C	C	E	P	T
↓ +1	↓ +2	↓ +3	↓ +4	↓ +5	↓ +6
B	E	F	I	U	Z

Shortcut : Both words 'REJECT' as well as 'ACCEPT' contain 6 alphabets, where the position of the second E and T in REJECT are same as the E and T in 'ACCEPT' i.e., positions (4) & (6) respectively.

So, the code should have I and Z at positions (4) & (6) respectively. Thus options (a) and (b) are ruled out. Also, since the code has alphabets moved up in an increasing order. Therefore, option (c) is invalid.

Hence option (d) is the correct answer.

Example 9 It in a certain language CHENNAI is coded as 'DGFMOZJ' how is MUMBAI coded in the same language?

- (a) NTNABH (b) LVL CZJ
(c) LILCBH (d) NVNCBJ

Solutions. (a) It can be clearly seen that the original word is moved 1 letter forward and backward alternately, such that

C	H	E	N	N	A	I
↓ +1	↓ -1	↓ +1	↓ -1	↓ +1	↓ -1	↓ +1
D	G	F	M	O	Z	J

Similarly, in case of the word MUMBAI, the alphabets will also be moved 1 letter forward & backward alternately therefore, options (b) & (c) are ruled out.

M	U	M	B	A	I
↓ +1	↓ -1	↓ +1	↓ -1	↓ +1	↓ -1
N	T	N	A	B	H

Hence, option (a) is the correct answer.

IV. Alpha-Numeric Series Coding: Out of the entire alphabet series numbered 1 to 26, i.e.,

A	B	C	D	E	F	G	H	I	J	K
1	2	3	4	5	6	7	8	9	10	11
L	M	N	O	P	Q	R	S	T	U	
12	13	14	15	16	17	18	19	20	21	
V	W	X	Y	Z						
22	23	24	25	26						

it is difficult to remember each and every alphabets' numeric value, therefore, we just have to remember.

E = 5, J = 10, O = 15, T = 20 and Y = 25 which are multiples of 5.

In alphanumeric coding, the alphabets of the word are given and we code them in terms of their numeric value or some pattern according to their numeric values.

Example 10 If in a certain code language, 'MIRROR' is coded as '139181815', how will 'APPLE' be coded in the same language?

- (a) 11616125 (b) 3984145
(c) 1162254 (d) 11213147

Solutions. (a) As we can see from the alphabet series, numeric values for the alphabets M is 13, I is 9, R is 18 and O is 15. Similarly, for the word APPLE, A = 1, P = 16, P = 16, L = 12 & E = 5. So, the code is 11616125. Therefore, option (a) is the correct answer.

Example 11 If EAT is 26, ZEAL is 44 and AROMA is 48.

- (1) How is 'HELMET' written in the same language?
(a) 23 (b) 63
(c) 83 (d) 53
(2) How is 'MANAGER' written in the same language?
(a) 89 (b) 59
(c) 79 (d) 39

Solutions. As we can see from the alphabet series, the words

EAT → E = 5, A = 1, T = 20

5 + 1 + 20 = 26

ZEAL → Z = 26, E = 5, A = 1, L = 12

26 + 5 + 1 + 12 = 44

AROMA → A = 1, R = 18, O = 15, M = 13, A = 1

1 + 18 + 15 + 13 + 1 = 48

(1) (b) similarly, HELMET is written as

HELMET → H = 8, E = 5, L = 12, M = 13, E = 5, T = 20

8 + 5 + 12 + 13 + 5 + 20 = 63

Hence, the code for 'HELMET' is 63.

option (c) is the correct answer.

(2) (b) similarly the word 'MANAGER' can be written as

MANAGER → M = 13, A = 1, N = 14, A = 1, G = 7,

E = 5, R = 18

13 + 1 + 14 + 1 + 7 + 5 + 18 = 59

so, the code for the word 'MANAGER' is 59.

option (c) is the correct answer.

Example 12 If in a certain language, 'IMAGE' is written as '571319', then 'SMALL' will be written as

- (a) 212113191 (b) 211211391
(c) 191311212 (d) 39421919

Solutions. (a) The code for 'IMAGE', according to the alphabet series is '913175'. When we reverse the order of this code, it is written as '571319' which is the given code.

Similarly, for the word 'SMALL', the code according to the alphabet series is '191311212'. When we reverse the order of this code, we obtain '212113191', which is the required code. Thus, option (a) is the correct answer.

DECODING

The word decoding stands for converting a certain pattern or expressions, i.e., the code, to a word from English language or a certain series of numbers.

In other words, decoding refers to the process of converting the code back to the original word.

Similar to coding, there are 4 types of Decoding Methods :

- I Simple Arrangement
- II Direct Substitution
- III Pattern Substitution
- IV Alphanumeric Decoding

We would first like you to go back and revise the four types of CODING methods before we move on.

I Simple Arrangement Method : Under this, the code will be obtained simply by re-arrangement of the alphabets of the word.

The questions will test you on decoding these codes.

Example 1 In a certain language, 'SIMPLE' is written as 'ISPMEI' and 'CHAPTER' is written as 'HCPARET'. Then 'LFWORE' stands for which word?

- (a) LOWFER (b) FLOREW
(c) FLOWER (d) WORFEL

Solutions. (c) SIMPLE $\rightarrow$ ISPMEI

The word 'SIMPLE' is of 6 letters and so is the code given in the question. Therefore, we have to follow the pattern of the word 'SIMPLE'.

In the above code, we can clearly observe that the code is obtained simply by inter changing the positions of consecutive alphabets, i.e.,

S	I	M	P	L	E
I	S	P	M	E	L

Similarly,

F	L	O	W	E	R
L	F	W	O	R	E

Hence, option (c) is the correct answer. 'FLOWER' is the word for which the code is given.

Example 2 Using the data from example 1, which word is coded 'ETCAREH'?

- (a) TEACHER (b) TEACREH
(c) EACHTER (d) TECAREH

Solutions. (a) The word 'CHAPTER' is of 7 letters & so is the code given in the question. Therefore, we have to follow the pattern of the word 'CHAPTER'.

C	H	A	P	T	E	R
H	C	P	A	R	E	T

Similarly,

T	E	A	C	H	E	R
E	T	C	A	R	E	H

Hence, option (a) is the correct answer. 'TEACHER' is the word for which the code is given.

Example 3 In a certain language, if 'CARROM' is written as 'MORRAC', then what is the word coded as 'TIBBAR'?

- (a) RIBBAT (b) RABBIT
(c) BARTIB (d) BITRAB

Solutions. (b) The code for the word 'CARROM', is obtained by reversing the order of the alphabets of the word i.e.,

C A R R O M
← reverse the order

M O R R A C

Similarly, the word for code 'TIBBAR' will be an outcome of the same pattern as followed for the word 'CARROM', i.e., the code TIBBAR will also be written in reverse order.

T I B B A R
← reverse the order

R A B B I T

Hence, 'RABBIT' is the word, making option (b) the correct answer.

Note for Decoding : Solve by options, i.e., by taking one option at a time into consideration.

Example 4 Using the data of eg. 3, which word would be coded as ELYTSEFIL?

- (a) LIFESTYLE (b) LIFEELYTS
(c) LIFESTYEL (d) EFLSTYLE

Solutions. (a) Since, the code for 'CARROM' was obtained by writing the letters in the reverse order,

C A R R O M
← reverse the order

M O R R A C

Similarly, the code when written in the reverse order will give us the original word, i.e.,

ELYTSEFIL

← reverse the order

LIFESTYLE

Hence, the word for which the code is given is LIFESTYLE.
Thus option (a) is the correct answer.

- II. Direct Substitution Method :** In these kind of questions, there will be one or more words given in the question for which codes will be given either in direct fashion or in jumbled up fashion. We'll discuss both.
We will be asked to find out the original word (s) for the given code(s).

Command data for Example 5 to Example 8 :

In a certain code language, if POURING is written as xfnplom, SAMPLE is written as zehxcj and WHITENER is written as atlkjojp.

Example 5 Then which word is written as 'hjecz' ?

- (a) LEAMS (b) SMEAL
(c) MEALS (d) MALES

Solutins. (c) The codes for the three words are direct substitution of the small alphabets for the capital ones, because 'N' & 'T' are codes as 'o' & 'l' respectively in both POURING as well as WHITENER, while 'E' is written as 'j' both the times in the word WHITENER and also in the word SAMPLE.

Words	P	O	U	R	I	N	G	S	A	M	L	E	W	H	T
Codes	x	f	n	p	l	o	m	z	e	h	c	j	a	t	k

Letter	P	O	U	R	I	N	G	S	A	M	L	E	W	H	T
Codes :	x	f	n	p	l	o	m	z	e	h	c	j	a	t	k

Therefore, the code 'hjecz' stands for MEALS.

MEALS
↑↑↑↑↑
h j e c z

deriving the word from the code (see arrows)

Hence, option (c) is the correct answer.

Example 6 Which word is coded as 'ajlmtk' ?

- (a) WAISTE (b) WEIGHT
(c) WASTES (d) HEIGHT

Solutins. (b) Following the same concept as in example 5, the code 'ajlmtk' represents

WEIGHT
↑↑↑↑↑
a j l m t k

deriving the word from the code.

Hence, option (b) 'WEIGHT' is the correct answer.

Example 7 Which word is coded as 'znhhjp' ?

- (a) SUMMER (b) WINTER
(c) WHITER (d) POORER

Solutins. (a) Since, all the words in the options start with different letters, therefore, their codes also start with different letters.

Thus we can see that the first letter of the code 'z' is the code for letter 'S'. Therefore, the given code stands for the word 'SUMMER'.

Hence, option (a) is the answer.

Example 8 Which word is coded as 'pjexlom' ?

- (a) PEERING (b) REEPING
(c) PEARING (d) REAPING

Solutins. (d) Similar to example 7's solution, we check for the first letter, where the code 'p' stands for the alphabet 'R'. Therefore, options (a) & (c) are ruled out and the answer has to be either option (b) or option (d). Since, the code has no repetition of alphabets, so option (b) is also invalid. Thus the correct answer is option (d) REAPING.

Common Data for Example (9 to 12) : In a certain code, "WOMAN" is coded as "DPCZR", "MASTER" is coded as "PCMEGX", "FLOWER" is coded as "BIGXDZ" and "FLOAT" is coded as "DCIEB", then in that language find the word for the following codes :

Example 9 IBDZ :

- (a) FLOW (b) BLOW
(c) WARE (d) WORE

Solutins. (a) Since, the letter 'O' seems to have different codes at its positions in the three words WOMAN, FLOWER and FLOAT, therefore, is not a case of direct substitution but of jumbled substitution (the question becomes tricky).

WORD	CODE
W O M A N	→ D P C Z R
M A S T E R	→ P C M E G X
F L O W E R	→ B I G X D Z
F L O A T	→ D C I E B

There are 3 words – WOMAN, FLOWER & FLOAT that have 1 same letter, i.e., 'O' for which they have one common code letter, i.e., 'D'. Similarly the words - WOMAN & LOWER have 1 more common 'W', for which they have one more common code letter 'Z'.

Similarly, the words – WOMAN, MASTER & FLOAT have a letter 'A' common in all these words for which the common code is 'C'.

Following the same trend

Letter	W	O	M	A	N	S	T	E	R	F	L
Code	Z	D	P	C	R	M	E	G/X	X/G	I	B

In the above table, the codes for the letters 'E' & 'R' cannot be found distinctly.

From the table, the code IBDZ stands for the word FLOW.

I	B	D	Z
↓	↓	↓	↓
F	L	O	W

Hence, option (a) is the correct answer.

QUICKTIPS

- Spot out the common letters in the words and try to find the code for the common letters first.
- As soon as you find the codes for some letters, tick those letters as well as codes so you don't keep checking them again & again. This will save your time & save you from any kind of confusion while solving the questions.

Example 10 ZGCX:

- (a) WORE (b) WEAR
(c) MERE (d) WERE

Solutions. (b) For the code letters Z, G, C, X the alphabets are

Z	G	C	X
↓	↓	↓	↓
W	E/R	A	E/R

Therefore, the word from the given options is 'WEAR'.
Correct option is (b).

Example 11 PCEEGX:

- (a) FARMER (b) WELLER
(c) FAMMER (d) MATTER

Solutions. (d) For the code letters PCEEGX, there has to be one alphabet in the word that is repeatedly used, thus option (a) is ruled out.

Now, the letters of the code stand for :

P	C	E	E	G	X
↓	↓	↓	↓	↓	↓
M	A	T	T	E/R	E/R

Hence, the word is 'MATTER', option (d).

Example 12 PCRX:

- (a) MANE (b) MEAN
(c) MARN (d) Cannot be determined.

Solutions. (d) For the code alphabets, the representation is

P	C	R	X
↓	↓	↓	↓
M	A	N	E/R

Since, it is not clear whether 'X' stands for 'E' or 'R', we cannot say clearly which word is correct. Hence, answer is option (d).

III. Pattern Substitution Method

Under this method, a code will be given to you, you will have to recognize the pattern the code is following. The pattern may be moving a few alphabets forward, a few alphabets backward or alternate forward & backward.

Example 13 If, in a certain language, POWERFUL is coded as QQZIWLB T, then which word is coded as ECQGJXZ ?

- (a) DANCERS (b) HARMLESS
(c) PRACTISE (d) DANGERS

Solutions. (a) The pattern followed by the code is moving up in an increasing order.

P	O	W	E	R	F	U	L
↓ +1	↓ +2	↓ +3	↓ +4	↓ +5	↓ +6	↓ +7	↓ +8
Q	Q	Z	I	W	L	B	T

First method : similarly, the code given will also follow the same pattern. Therefore, we subtract or move the alphabets of the code backwards to form the word.

D	A	N	C	E	R	S
↑ -1	↑ -2	↑ -3	↑ -4	↑ -5	↑ -6	↑ -7
E	C	Q	G	J	X	Z

Second method : We can also go by the options, i.e., check each option one by one and see if it forms the code given in the questions using the same pattern as coded in the word 'POWERFUL'.

IV. Alpha-Numeric Decoding

This is similar to alpha-numeric coding, with the only difference that we have to derive the code from a given word in case of alpha-numeric coding. While in decoding, we have to derive the word from the code following the alphabet series (ABCDEF G PQRSTU VWXYZ).

Example 14 If the code in a certain language, for PAPER = 56 and SHEET = 57, then for which of the following words is the code 88 ?

- (a) IRON (b) PUPPET
(c) HELMETS (d) PARROT

Solutions. (d) As we know the values of each alphabet in the alphabet series, so we can unlock the codes.

PAPER → P = 16, A = 1, P = 16, E = 5, R = 18
16 + 1 + 16 + 5 + 18 = 56

SHEET → S = 19, H = 8, E = 5, E = 5, T = 20
19 + 8 + 5 + 5 + 20 = 57

Now, to find the word for the code 88, we have to check all the options.

Option (a)

IRON → I = 9, R = 18, O = 15, N = 14
9 + 18 + 15 + 14 = 56

Option (b)

PUPPET → P = 16, U = 21, P = 16, P = 16, E = 5, T = 20,
16 + 21 + 16 + 16 + 5 + 20 = 94

Chapter 02: Blood Relations

MEANING OF BLOOD RELATION

Blood relation does mean biological relation. Remember a wife and husband are not biologically related but they are biological parents of their own children. Similarly, brother, sister, paternal grandfather, paternal grandmother, maternal grandfather, maternal grandmother, grandson, granddaughter, niece, cousin etc. are our blood relatives.

TYPES OF BLOOD RELATIONS

There are mainly two types of blood relatives:

- (i) Blood relation from paternal side
- (ii) Blood relation from maternal side

Now, we will discuss both kind of relations one-by-one.

- (i) **Blood relation from paternal side** : This type of blood relation can be further subdivided into three types:

- (a) **Past generations of father** : Great grandfather, great grandmother, grandfather, grandmother etc.
- (b) **Parallel generations of father**: Uncles (Brothers of father), aunts (sisters of father) etc.
- (c) **Future generations of father**: Sons, daughters, grandsons, granddaughters etc.

- (ii) **Blood relation from maternal side**: This type of blood relations can also be subdivided into three types:

- (a) **Past generations of mother**: Maternal great grandfather, maternal great grandmother, maternal grandfather, maternal grandmother etc.
- (b) **Parallel generations of mother**: Maternal uncles, maternal aunts etc.
- (c) **Future generations of mother**: Sons, daughters, grandsons, granddaughters etc.

In the examination, the questions are given in complicated way. In other words, in the given questions, the easy relationship takes the complicated form and examinees are expected to solve this complication in order to find out the correct answer. How does an examinee get aid of this complication? For this, an examinee sees the given data in the question with a serious eye; then tries to establish relation among elements of given data on the basis of certain logic and finally finds out the required answer. In fact complications in the asked question occur because of the given indirect relation. It does mean questions are in the form of indirect relation & one has to convert this indirect relation into direct relation. For example "only son of my father" does mean 'me' (myself). Here in place of 'me' indirect relation has been given in form of "only son of my father". Similarly, "the only

daughter of the parents in laws of the husband of Vandana" does mean 'Vandana' herself. In this example also the sentence "the only daughter of the parents in laws of the husband of 'Vandana' has been given in the form of indirect relation. Below are given some indirect relation in the form of a list. Examinees are required to learn them by heart. If one keeps this list in one's mind, he/she will find it very easy to solve problems based on blood relations.

- | | | |
|------------------------------------|---|-----------------------------|
| 1. Son of father or mother | : | Brother |
| 2. Daughter of father or mother | : | Sister |
| 3. Brother of father | : | Uncle |
| 4. Brother of mother | : | Maternal uncle |
| 5. Sister of father | : | Aunt |
| 6. Sister of mother | : | Aunt |
| 7. Father of father | : | Grandfather |
| 8. Father of father's father | : | Great grand father |
| 9. Father of grandfather | : | Great grandfather |
| 10. Mother of father | : | Grandmother |
| 11. Mother of father's mother | : | Great grandmother |
| 12. Mother of grandmother | : | Great grandmother |
| 13. Father of mother | : | Maternal grandfather |
| 14. Father of mother's father | : | Great maternal grand father |
| 15. Father of maternal grandfather | : | Great maternal grandfather |
| 16. Mother of mother | : | Maternal grandmother |
| 17. Mother of mother's mother | : | Great maternal grandmother |
| 18. Mother of maternal grandmother | : | Great maternal grandmother |
| 19. Wife of father | : | Mother |
| 20. Husband of mother | : | Father |
| 21. Wife of Grandfather | : | Grandmother |
| 22. Husband of Grandmother | : | Grandfather |
| 23. Wife of son | : | Daughter-in-law |
| 24. Husband of daughter | : | Son-in-law |
| 25. Brother of Husband | : | Brother-in-law |
| 26. Brother of wife | : | Brother-in-law |

27. Sister of Husband	:	Sister-in-law
28. Sister of wife	:	Sister-in-law
29. Son of brother	:	Nephew
30. Daughter of brother	:	Niece
31. Wife of brother	:	Sister-in-law
32. Husband of sister	:	Brother-in-law
33. Son of sister	:	Nephew
34. Daughter of sister	:	Niece
35. Wife of uncle	:	Aunt
36. Wife of maternal uncle	:	Aunt
37. Son/daughter of uncle/Aunt	:	Cousin
38. Son/daughter of maternal uncle/maternal aunt	:	Cousin
39. Son/daughter of sister of Father	:	Cousin
40. Son/daughter of sister of Mother	:	Cousin
41. Only son of grandfather	:	Father
42. Only daughter of maternal grandfather	:	Mother
43. Daughter of grandfather	:	Aunt
44. Sons of grandfather other than father	:	Uncle
45. Son of maternal grandfather /maternal grand mother	:	Maternal Uncle.
46. Only daughter in law of grandfather/ grandmother	:	Mother
47. Daughters in law of grandfather/ grandmother	:	Aunt other than mother
48. Daughters-in-law of maternal grandfather/ grandmother	:	Aunt maternal
49. Neither brother nor sister	:	Self

SOME IMPORTANT INFORMATION ABOUT BLOOD RELATION

- A. Without the information of gender, no relationship can be established between two people. For example, If given that R is the child of P & Q, then we can only say that P & Q are the parents of R. But we can not find out:
- R is the son of P & Q or R is the daughter of P & Q.
 - Who is mother of R and who is father of R.
- But if we have given that P is a male, Q is a female and R is male, then we can easily say that R is the son of P and Q. Further we can also say that P is father of R and Q is mother of R.
- B. Gender can not be decided on the basis of name. For example in Sikh community the names like Manjit, Sukhvinder etc. are the names of both male and female. Similarly, in the Hindu Community 'Suman' is the name of both male and female.

Remember: Solution Tips

- While solving blood relation based question, first of all find out that two persons between whom a relationship has to be established.
- Next, try to find out middle relation
- Finally find out the relationship between two persons to be identified for this purpose.

Type of problems

- General problems of blood relation
- Blood relation based on family tree
- Coded blood relationship.

Now, we will discuss all the three types of problems one by one

(1) General problem of blood relation

Sample Q: Pointing towards a photograph, Mr. Sharma said, "She is the only daughter of mother of my brother's sister." How is Mr. Sharma related to the lady in the photograph?

- Cousin
- Sister
- Aunt
- Daughter in law
- None of these

Ans. Here we have to find relationship between Mr. Sharma & the lady in the photograph.

Mother of my brother's sister does mean my (Mr. Sharma's) mother. Only daughter of Mr. Sharma's mother does mean "sister of Mr. Sharma". Hence option (b) is the correct answer.

(2) Blood relation based on family tree

Sample Q: Q is the brother of C and C is the sister of Q. R and D are brother and sister. R is the son of A while A & C are wife and husband. How is Q related with D.

Ans. For such type of question a family tree is made in which some symbols are used as below:

' $\longleftrightarrow$ ' is used for husband & wife.

' --- ' is used for brother & sister

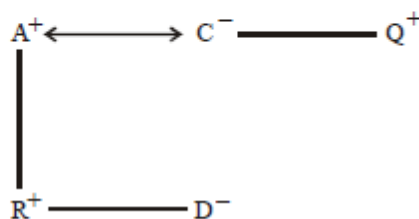
' $|$ ' is used for parents (father or mother). Parents are put on top while children are put at the bottom.

' $-$ ' or minus sign is used for female

' $+$ ' or plus sign is used for male.

Now adopting and using the above given symbols we can make a family tree and solve the given problem, let us see the family tree for sample question:

Family+ tree :



As per the question Q is the brother of C and C is the sister of Q. Hence relation between C & Q has been presented as ($C^- \text{---} Q^+$) where ' $-$ ' sign above C makes it clear that C is a female and ' $+$ ' sign above 'Q' makes it clear that Q is a male. Similarly for R and D. The presentation ($R^+ \text{---} D^-$) has been made. Further according to the question.

A and C are having a husband and wife relationship and hence this has been presented as $\left(\overset{+}{A} \rightleftharpoons C^- \right)$. As it is already given that C is the sister of Q and A and C are wife and husband, this becomes clear that A is the male member of the family and this is the reason A has '+' as its gender sign. Lastly, the vertical line gives father

and son relationship and has been presented as $\left(\begin{array}{c} A^+ \\ | \\ R^+ \end{array} \right)$. Now from this family tree it becomes clear that C is the mother of R and D and as Q is the brother of C, then Q will definitely be the maternal uncle of R & D. Hence we can say that Q is the maternal uncle of D and this is the required answer for our sample question.

NOTE : Sample problem is the very easy type of blood relation problem. In this chapter we will solve only such type of problem based on family tree. But we will solve more difficult problems of such type in a different chapter "Problem solving".

(3) Coded blood relationship.

Sample: Directions: Read the following informations carefully to give the answers of following questions:

'P × Q' means P is the brother of Q

'P - Q' means P is the sister of Q

'P + Q' means P is the father of Q

'P ÷ Q' means P is the mother of Q.

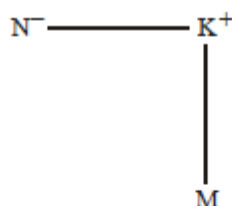
Which of the following option is the presentation of M is the nephew of N?

- (1) N - K + M
- (2) N × K ÷ M
- (3) N ÷ K × M
- (4) N - K + M × T
- (5) None of these.

Solution: To solve it we will use the symbols of family tree in place of mathematical signs (+, -, × & ÷). Let us make family tree presentation for every option:

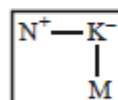
Presentation for option (1) [N - K + M]

Here gender of M can not be find out so, this option is rejected, point to be noted that even without making a family tree. You



can find out that this option can not give you the gender of M. For this only a serious look at the option is enough.

Presentation of option (2) [N × K ÷ M]

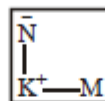


This family tree presentation also does not give the

gender of M. Like option (1) this option gives you a clear indication, only by a serious look, that gender of M can not be find out and for this making family tree is not necessary. Hence option (2) is also rejected.

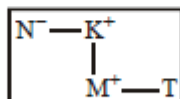
Presentation of option (3) [N ÷ K × M]

Like option (1) and (2), option (3) is also rejected and only a serious look can make you clear that in this case also the gender of M can not be find out



Presentation of option (4) [N - K + M × T]

This presentation makes it clear that N is the sister of K who is father of M. Here gender of M is clear that M is a male. Hence, M is clearly nephew of N.



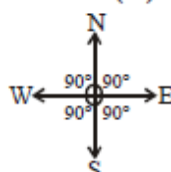
Hence for sample question option (4) is the correct answer. Now we have come to the conclusion of this chapter. Readers are advised to practice more and more to crack such questions quick as possible.

Chapter 03: Directions

MAIN AND CARDINAL DIRECTIONS

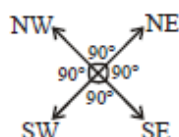
Main Directions

1. North (N)
2. South (S)
3. East (E)
4. West (W)



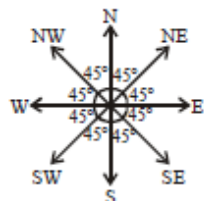
Cardinal Directions

1. North East (NE)
2. North West (NW)
3. South East (SE)
4. South West (SW)



Angle between any two Main or Cardinal Directions

Angle between any two main (or cardinal) directions is 90° but the angle between one main and one cardinal direction is 45° .



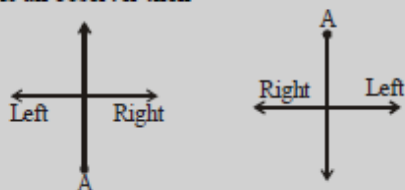
Sign for showing direction – $\uparrow$.

Here, the apex of an arrow tells us about the direction of movement of a point and the tail of an arrow shows the beginning of that point.

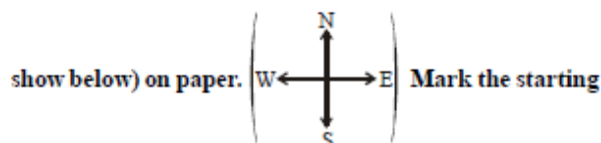
Clockwise direction (↻): When a point moves in the direction of hands of clock, then it can be said that the point is moving in CW direction.

Anticlockwise direction (↺): When a point moves in the opposite direction of hands of clock, then it can be said that the point is moving in ACW direction.

If 'A' is an observer then



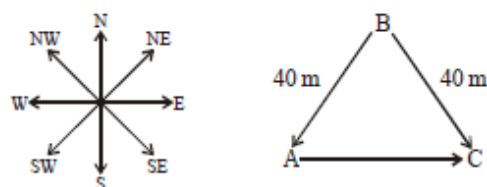
To solve the question, first draw the direction figure (as



point. After that move carefully according to the directions given in the question.

Example 1 A is 40 m south-west of B. C is 40 m south-east of B. Then, C is in which direction of A?

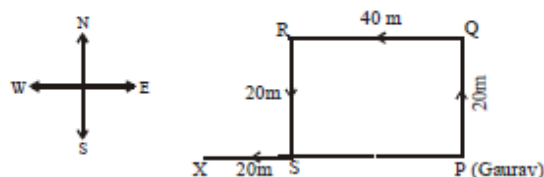
Solution : As clear from the adjoining diagram, C lies to the east of A.



Example 2 Gaurav walks 20 metres towards North. He then turns left and walks 40 metres. He again turns left and walks 20 metres. Further, he moves 20 metres after turning to the right. How far is he from his original position?

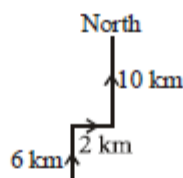
Solution : The movements for Gaurav are as shown in figure.

Clearly, Gaurav's distance from his initial position $P = PX = (PS + SX) = (QR + SX) = (40 + 20) \text{ m} = 60 \text{ m}$.



Example 3 After walking 6 km, I turned right and covered a distance of 2 km, then turned left and covered a distance of 10 km. In the end, I was moving towards the north. From which direction did I start my journey?

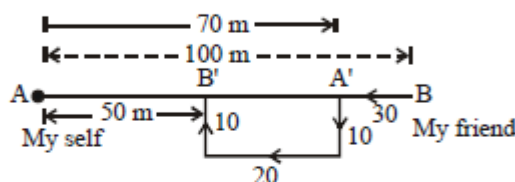
Solution : The route is as shown in the adjoining diagram.



Thus, the man started his journey from the south and moved northwards.

Example 4 My friend and I started walking simultaneously towards each other from two places 100 m apart. After walking 30 m, my friend turns left and goes 10 m, then he turns right and goes 20 m and then turns right again and comes back to the road on which he had started walking. If we walk with the same speed, what is the distance between us at this point of time?

Solution :



When my friend reaches on the previous track (i.e. on B') again, he had travelled a distance of $(30 + 10 + 20 + 10) = 70$ m. As I walk with the same speed as that of my friend I have walked 70 m, but on the straight track. Now, he is just $[100 - (30 + 20)] = 50$ m from my starting point.

Hence, the distance between us = $(70 - 50) = 20$ m

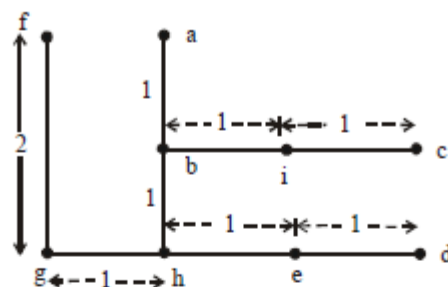
Example 5-7 : Read the information given below to answer these questions.

a, b, c, d, e, f, g, h and i are nine houses. c is 2 km east of b. a is 1 km north of b and h is 2 km south of a. g is 1 km west of h while d is 3 km east of g and f is 2 km north of g. i is situated just in the middle of b and c while e is just in middle of h and d.

5. Distance between e and g is :
 (a) 2 km (b) 1 km
 (c) 5 km (d) 1.5 km
6. Distance between a and f is :
 (a) 1.41 km (b) 3 km
 (c) 2 km (d) 1 km
7. Distance between e and i is :
 (a) 4 km (b) 2 km
 (c) 1 km (d) 3 km

Solutions 5 to 7 :

From the information given, positions of houses are as follows :

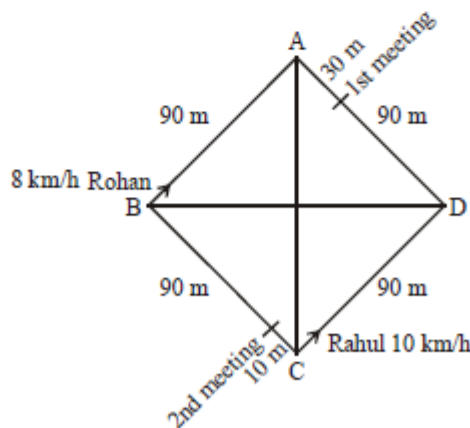


5. (a) Clearly, the distance between e and g is 2 km.
6. (d) From the above diagram, the distance between a and f is 1 km.
7. (c) Clearly, the distance between e and i is 1 km.

Example 8 A square field ABCD of side 90 m is so located that its diagonal AC is from north to south and the corner B is to the west of D. Rohan and Rahul start walking along the sides from B and C respectively in the clockwise and anti-clockwise directions with speeds of 8 km/hr and 10 km/hr. Where will they cross each other the second time ?

- (a) On AD at a distance of 30 m from A
- (b) On BC at a distance of 10 m from B
- (c) On AD at a distance of 30 m from D
- (d) On BC at a distance of 10 m from C

Solution : (d)



Speeds of Rohan and Rahul are in the ratio 4 : 5.

Chapter 04: Venn diagram

VENN DIAGRAM

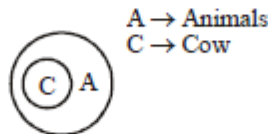
Venn diagram is a way of representing sets pictorially. This idea was developed by John Venn, an English Mathematician.

SET : A set is a collection of well-defined objects, called the elements of the set.

Representation of different sets

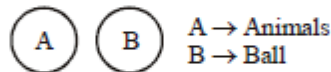
- (A) **Subset :** If one kind of item is completely included by the other.

For Example : Animals and Cow



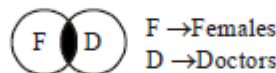
- (B) **Disjoint sets :** If the items are completely exclusive of the other.

For Example : Animals and Ball



- (C) **Intersection of two sets :** If one kind of item is partially included in other one.

For Example : Doctors and Females

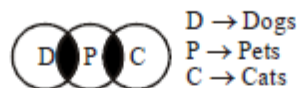


Some females are doctors.

Some doctors are females.

- (D) **Intersection of three sets :** If items are partially included in others.

For Example : Dog, Pets and Cats



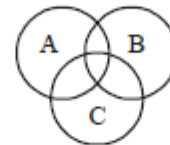
Some dogs and cats are pets; not all.

Some pets are neither dogs nor cats.

- (E) **If the three items are partly related to each other :**
They are represented as shown in the adjoining figure.

For Example :

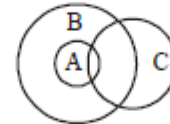
Clerks, Government Employees and Educated Persons
Clearly, some clerks may be government employees and some may be educated. Similarly, some government employees may be clerks and some may be educated. Also, some educated persons may be clerks and some may be government employees. So, the given items may be represented as shown in the following figure with three different circles denoting the three classes.



- (F) **If one item belongs to the class of second and the third item is partly related to these two :** They are represented as shown alongside.

For Example : Females, Mothers and Doctors

Clearly, all mothers are females. This would be represented by two concentric circles. But, some females and some mothers can be doctors. So, the circle representing doctors would intersect the two concentric circles. Thus, the diagram becomes as shown in the following figure with circle A representing Mothers, circle B representing Females and circle C representing Doctors.

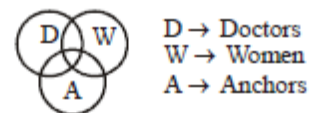


Example 1

Diagrammatic representation of the following :

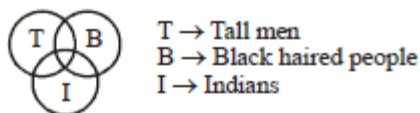
- (i) Women, Doctors, Anchors
- (ii) Tall men, Black haired people, Indians
- (iii) Mars, Solar System, Universe
- (iv) Girls, Boys, Students
- (v) Children, Men, Women

Solution : (i)



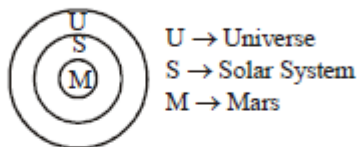
Some women are doctors and some are anchors. Among these women, some are both doctors and anchors.

(ii)



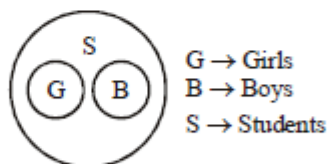
Some Indians are tall men. Some Indians have black hair. Among these Indians, some Indians are tall men with black hair.

(iii)



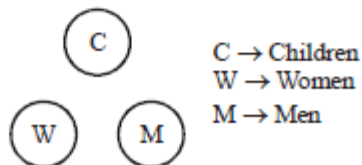
Universe contains Solar System and Solar System contains Mars.

(iv)



Girls and boys are students.

(v)

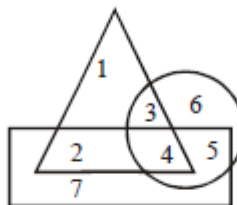


Children, Women and Men are three different groups.

Some times venn diagram can be represented by different types of geometrical figures.

Example 2

'Triangle' represents young persons, 'circle' represents uneducated persons and 'rectangle' represents employed persons.



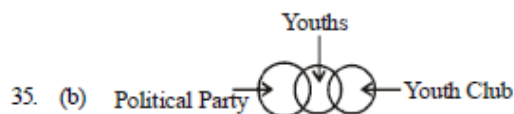
Give the answers of the following questions :

- Which region represents young, uneducated and employed persons?
- Which region represents educated, employed and young persons?
- Which region represents young, educated and unemployed persons?
- Which region represents young, uneducated and unemployed persons?

Solution :

- Region - 4, because this region is common to all three.
- Region - 2, because this region is common between young persons and employed persons but not with uneducated.
- Region - 1, because this region contains only young persons not a part of uneducated region and employed region.
- Region - 3, because this region is common between young persons and uneducated persons but not included in employed region.

31. (b) The required region should be common to circle and square and outside the triangle. Such region is marked '2'.
32. (d) It is clear from the diagram 200 knew only Hindi.
33. (c) The required region should be only in circle. Such region is marked '9'.
34. (b) 11 persons speaks Kannada, $n(K) = 11$
 20 persons speaks Tamil, $n(T) = 20$
 11 persons speaks Telugu, $n(Te) = 11$
 2 persons speaks two languages
 $n(K \cap T) + n(T \cap Te) + n(K \cap Te) = 2n$
 1 persons speaks all the languages, $(K \cap T \cap Te) = 1$
 We have
 $n(K \cup T \cup Te) = n(K) + n(T) + n(Te) - [n(K \cap T) + n(T \cap Te) + n(K \cap Te)] + n(K \cap T \cap Te)$
 $= 11 + 20 + 11 - [2] + 1$
 $= 42 - 1 = 41$
 Total persons in the group is 41



Youths can be the member of both political party as well as youth club.

36. (b) Bonus and Dividend are different from each other. But both these are parts of profit.
37. (b) As ship is in the ocean in the same way sailor is in the ship.
38. (b) 17 persons take tea and wine but not coffee.
39. (c) Why women are not mentioned in the diagram?
 Ans. Rectangle represents men, therefore the area outside the rectangle should be WOMEN.
 How to find the answer for the this question?
 Ans = [Woman & Urban & Government_Employee]
 Conditions: Outside the rectangle(women) & circle (urban) & Square(Govt. employee)
 From the given diagram we can find that the value 10 is satisfies the above conditions.
 Therefore the answer is 10.
41. (c) Number of backward people who are not educated
 $= 17 + 5 = 22$.
42. (b) It is the common area between A and C but not including B, i.e., u.
43. (a) Both carrot and vegetable come under the food category, while carrot comes under vegetable category.
44. (a) As all species comes under the male category, so B and C comes under A. Some guitarists are bearded, so B and C have some common elements between them.

○○○

Chapter 05: Syllogism

SYLLOGISM

Syllogism is originally a word given by the Greeks. It means 'inference' or 'deduction'.

The problem of syllogisms can be solved mentally by using a little intelligence and common sense. Before we start it is necessary to understand the definition of certain terms.

Proposition

A proposition is a sentence that makes a statement and gives a relationship between two terms. It consists of three parts:

- (1) Subject
- (2) Predicate and
- (3) Copula

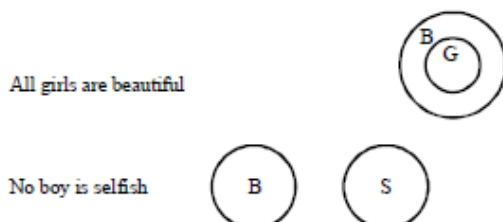
A subject is that part of the proposition about which something is being said. A predicate, on the other hand, is that term of the proposition which is stated about or related to the subject. A copula is a connecting word.

Categorical Proposition

A categorical proposition makes a direct assertion. It has no condition attached with it. There are two categorical propositions.

[A] Universal Proposition

Universal Proposition either fully include the subject or fully exclude it. Examples are:



Usually, universal propositions begin with "All", "Every", "Any", "No" etc. A proposition of the "All" type is universal positive proposition while a proposition of the latter is universal negative proposition. Universal positive proposition is denoted by "A" whereas universal negative proposition is denoted by "E".

[B] Particular Proposition

Particular Proposition either only partly includes or only partly excludes the subject while making a statement. Examples are:

Some rectangles are squares.

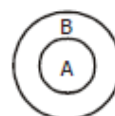
Some spheres are not circles.

A proposition of the former type are called particular positive proposition and is denoted by "I" whereas a proposition of the latter type are called particular negative proposition and is denoted by "O".

Euler's Circles or Venn Diagram

The best method of solving the Syllogism's problems is Euler's circle or venn diagram. There are four ways in which the relationship between the subject and predicate can be made.

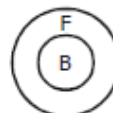
Type 1: 'All A are B' means that the whole circle denoting A lies within the circle denoting B



Example : All Lions are Animal

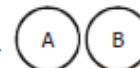


All Birds can Fly



Type 2: 'No A is B' means that circles denoting A and B do

not intersect at all.



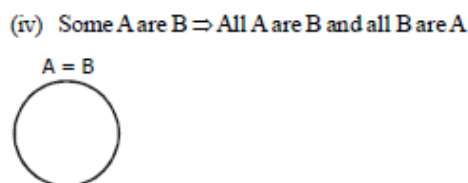
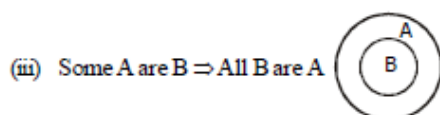
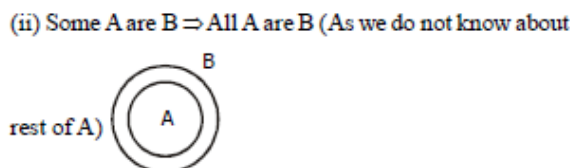
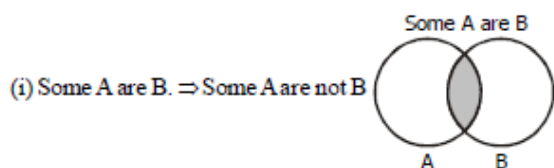
Example: No Man is Bad



No Man is Woman

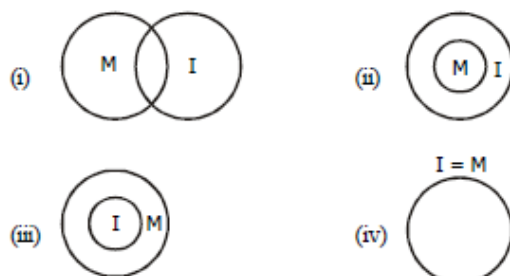


Type 3: 'Some A are B' it means that some part of the circle denoted by A is within the circle denoted by B. This type of statement gives two conclusions.

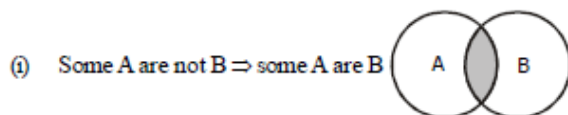


Example:

Some mosquitoes are insects



Type 4: 'Some A are not B', in this case some portion of circle A has no intersection with circle B while the remaining portion of circle A is uncertain whether this portion touches B or not. So this type of statement gives two conclusion.



(ii) Some A are not B $\Rightarrow$ No A is B.



Analytical Method

Try to understand these type of questions using analytical method.

A statement always has a subject and a predicate: All politicians

(subject)

are liars.

(predicate)

Basically, there are four types of sentences.

A – type $\Rightarrow$ All politicians are liars.

I – type $\Rightarrow$ Some politicians are liars.

O – type $\Rightarrow$ Some politicians are not liars.

E – type $\Rightarrow$ No politicians are liars.

Conclusions can be drawn by taking two of the above statements together. The rules of conclusion are:

A + A = A A + E = E I + A = I I + E = O
E + A = O* E + I = O*

Conclusion can only be drawn from the two statements if the predicate of the first statement is the subject of the second statement. The common term disappear in the conclusion and it consists of subject of the first statement and predicate of the second statement.

Relevance of O*: O* represent that the subject and the predicate are interchanged in the statement of conclusion.

Examples:

A + A = A

(i) All boys are girls.

(ii) All girls are healthy.

Conclusion: All boys are healthy.

A + E = E

(i) All boys are girls.

(ii) No girls are healthy.

Conclusion: No boys are healthy.

I + A = I

(i) Some boys are girls.

(ii) All girls are healthy.

Conclusion: Some boys are healthy.

I + E = O

(i) Some boys are girls.

(ii) No girls are healthy.

Conclusion: Some boys are not healthy.

E + A = O*

(i) No boys are girls.

(ii) All girls are healthy.

Conclusion: Some healthy are not boys.

E + I = O*

(i) No boys are girls.

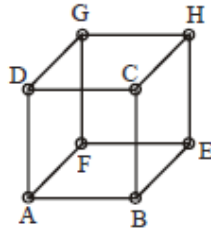
(ii) Some girls are healthy.

Conclusion: Some healthy are not boys.

Chapter 06: Cubes and Dices

What is a cube?

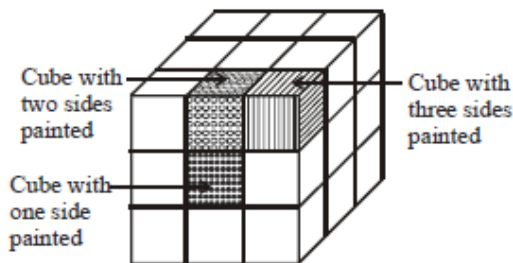
A cube is three dimensional figure whose length, breadth and height are equal and any two adjacent faces are inclined to each other at 90° . It has 6 faces, 8 corners and 12 edges.



- Corners of the cube are A, B, C, D, E, F, G and H.
- Edges of the cube are AB, BE, EF, AF, AD, CD, BC, EH, CH, GH, DG and FG.
- Faces of the cube are ABCD, EFGH, CDGH, BCHE, ABEF and ADFG.

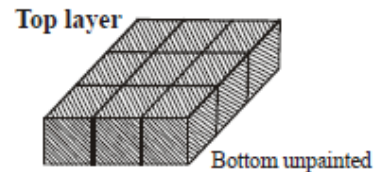
When a cube is painted on all of its faces with any colour and further divided into various smaller cubes of equal size, we get following results :

- (i) Smaller cubes with no face painted will present inside faces of the undivided cube.
- (ii) Smaller cubes with one face painted will present on the faces of the undivided cube.
- (iii) Smaller cubes with two faces painted will present on the edges of undivided cube.
- (iv) Smaller cubes with three faces painted will present on the corners of the undivided cube.

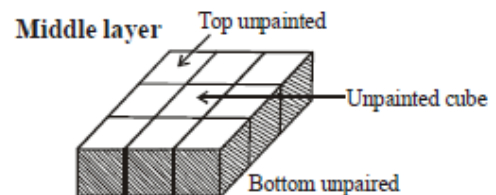


The above figure may be analysed by dividing it into three horizontal layers :

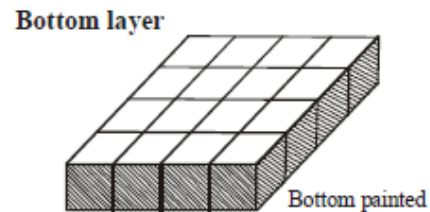
Layer I or Top Layer : The central cube has only one face coloured, four cubes at the corner have three faces coloured and the remaining 4 cubes have two faces coloured.



Layer II or middle layer : The central cube has no face coloured, the four cubes at the corner have two faces coloured and the remaining 4 cubes have only face coloured.



Layer III or bottom layer : The central cube has only one face coloured, four cubes at the corner have three faces coloured and the remaining 4 cubes have two faces coloured.



Also, if n = no. of divisions on the faces of cube

$$= \frac{\text{Length of the edge of undivided cube}}{\text{Length of the edge of one smaller cube}}$$

Chapter 07: Non- Verbal Reasoning

NON-VERBAL SERIES

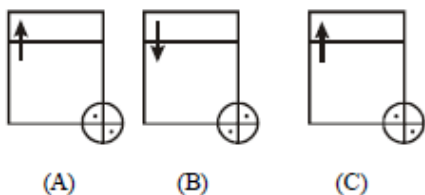
The word "series" is defined as anything that follows or forms a specific pattern or is in continuation of a given pattern or sequence.

In this type of nonverbal test, two sets of figures pose the problem. The sets are called problem Figures and Answer Figures. Each problem figure changes in design from the preceding one. The answer figure set contains 4 figures marked a, b, c, d. You are required to choose the correct answer figure which would best continue the series.

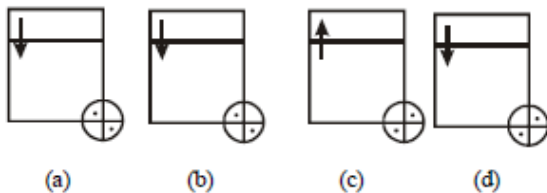
TYPE 1: A definite relationship between elements in given figures.

Example 1 Study the problem figures marked (A), (B) and (C) carefully and try to establish the relationship between them. From the answer figures marked a, b, c and d, pick out the figure which most appropriately completes the series.

Problem Figures



Answer Figures

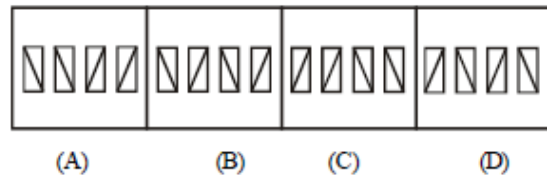


Solution :

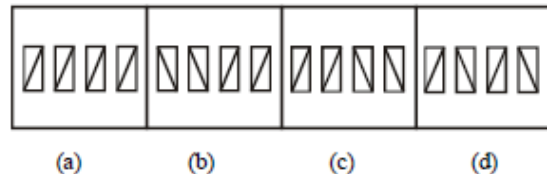
- (d) Note the direction of arrow which changes alternately. The dots are also changing alternately. Hence we are looking for a figure in which the arrow points down and the dots are positioned as in figure (B).

Example 2

Problem Figures



Answer Figures

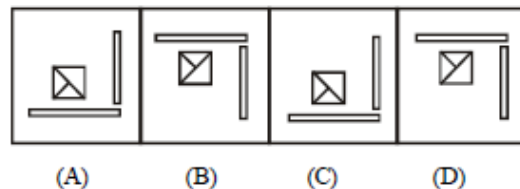


Solution :

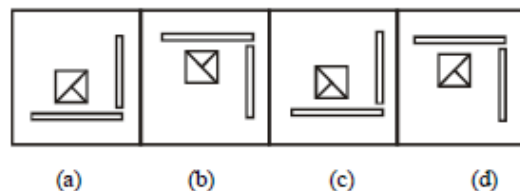
- (b) The four boxes are changing position in the following way : At first, middle boxes change position (diagonally) and extreme boxes remain stationary, then extreme boxes change position and middle boxes remain stationary and so on.

Example 3

Problem Figures

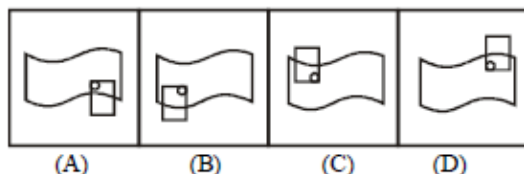
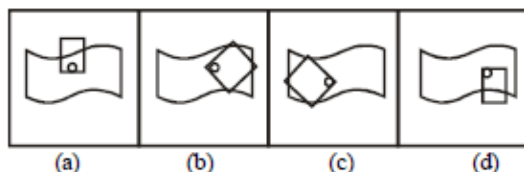


Answer Figures

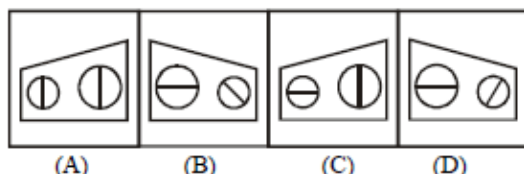
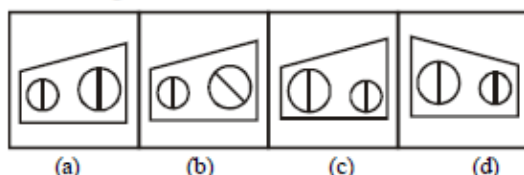


Solution :

- (c) The same figures rotates up-side-down in alternate figures.

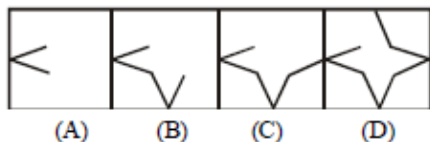
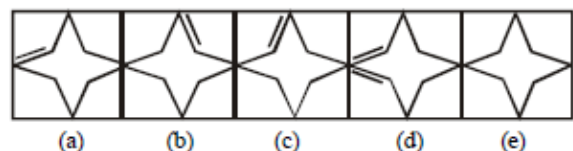
Example 4**Problem Figures****Answer Figures****Solution :**

- (d) The rectangle is shifted clockwise to edges of wavy figure and circle inside rectangle is kept to inside of wavy figure. So fifth figure in the series will be same as first figure.

Example 5**Problem Figures****Answer Figures****Solution :**

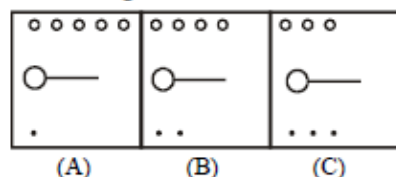
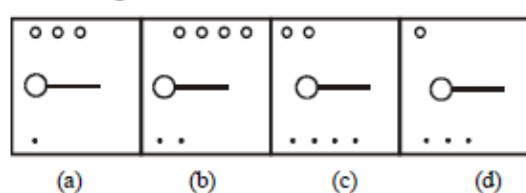
- (a) The bigger balls diameter is moving at 90° and smaller balls diameter is moving at 45° , simultaneously the face of the bigger figure is changing position.

TYPE 2: Additions of Elements : In these type of questions, each figure is obtained by either sustaining the element of preceding figure as it is or adding a part of element or one element or more than one element of the preceding figure in a systematic way.

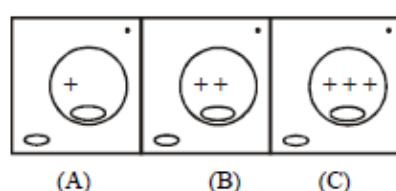
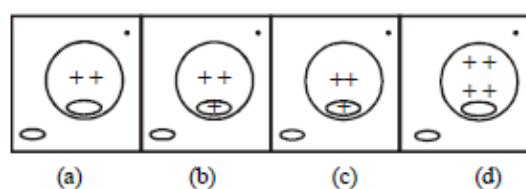
Example 6**Problem Figure****Answer Figure****Solution :**

- (e) Two line segments are added in P1 to obtain P2 and one line segment is added in P2 to obtain P3. This process is repeated again to obtain P4. Hence, answer figure 5 continues the series.

TYPE 3: In these questions the items in the diagrams either increase or decrease in number.

Example 7**Problem Figures****Answer Figures****Solution :**

- (c) The small circles are decreasing consecutively and the black dots are increasing.

Example 8**Problem Figures****Answer Figures****Solution :**

- (d) Signs of Plus are adding one by one. Figure (1) has one plus sign, Figure (2) has two signs, figure (3) has three signs, the next figure should have 4 signs to keep the same pattern.

Analytical Ability

Chapter 01: Statement and Assumptions

In this type of questions, a statement is given followed by two or more assumptions. We are required to assess the given statement and decide which of the given assumptions is implicit in the statement.

ASSUMPTION

An assumption is something that can be supposed or assumed on the basis of the given statement.

Some Tips on Assumptions :

(I) Key words

For evaluating an assumption, we can take a clue from some words that lend a definite meaning to the statement.

For example, 'all', 'only', 'each', 'every' etc. are definitive in nature whereas 'a few', 'some', 'many' etc. are not.

(II) Adjectives

An adjective is something which denotes a quality of the subject. If an adjective is attached to any subject, it must be assumed that the subject have the quality as denoted by the adjective.

Directions (for Examples 1 to 3) : In each of following questions, a statement is given followed by two assumptions numbered I and II. Consider the statement and the following assumptions and decide which of the assumptions is implicit. Give answer :

- (a) if only assumption I is implicit.
- (b) if only assumption II is implicit.
- (c) if either I or II is implicit.
- (d) if both I and II are implicit.
- (e) if neither I nor II is implicit

Example 1

Statement : India must earn a lot of foreign exchange to achieve her target of economic development.

Assumptions : I India desires to achieve the target of economic development.
II It is possible for India to earn more foreign exchange.

Solution :

- (d) I is implicit. Tools for an objective are talked about only when the desire for such an objective exists. II is implicit because it makes no sense to talk of something without the existence of its possibility.

Example 2

Statement : An advertisement : If you want to follow the footprints of an ideal leader, wear 'X' brand of shoes.

Assumptions : I Most people like to become ideal leaders.
II One can't become ideal leader unless one wears 'X' brand of shoes.

Solution :

- (a) I is implicit; that is why the advertisement has been given. Second one is absurd.

Example 3

Statement : Central Bank, which is the largest bank in the country, has decided to reduce its workforce by 30 per cent so that its banks may work efficiently.

Assumptions : I The Bank can perform all its activities after the reduction in workforce.
II The surplus employees may be asked to adopt early retirement scheme before leaving the bank.

Solution :

- (e) I is not implicit because of the word 'all'. II may or may not be a method of reducing the workforce. Hence, II is not implicit.

Directions (for Examples 4 to 6) : In each of following questions, a statement is given followed by two assumptions numbered I and II. Consider the statement and the following assumptions and decide which of the assumptions is implicit. Give answer :

- (a) if only assumption I is implicit.
- (b) if only assumption II is implicit.
- (c) if either I or II is implicit.
- (d) if neither I nor II is implicit.

Example 4

Statement : "Private Property, trespassers will be prosecuted" — A notice on a plot of land.

Assumptions : I The passerby may read the notice and may not trespass.
II The people are scared of prosecution and, therefore, never trespass.

Solution :

- (a) Whenever such notices are displayed it is assumed that those who are concerned with the notice will read the notice and follow the messages in it. Hence I is implicit. The notice has been placed making the intention of prosecution clear. But it cannot be said certainly. Hence II is not implicit.

Example 5

Statement : The nutritional status of children in India is better compared to that in other developing countries.

- Assumptions :** I It is not possible to estimate nutritional requirement of children in other countries.
II. India can become a developed country.

Solution :

- (d) Assumption I contradicts the statement. Nothing can be assumed about the scale of becoming developed. Hence II is not implicit.

Example 6

Statement : Please do not use lift while going down – an instruction on the top floor of a five-storey building.

- Assumptions :** I While going down, the lift is unable to carry any load.
II. Provision of lift is a matter of facility and not of right.

Solution :

- (d) Both I and II are not implicit as they are out of context. Nothing has been mentioned in the statement.

Chapter 02: Critical Reasoning

Types of Critical Reasoning Questions

Critical reasoning questions will ask you to:

1. Identify the inference / Must be true question
2. Identify the assumption.
3. Strengthen an argument.
4. Weaken an argument.
5. Select the best conclusion / Main Point
6. Identify the paradox
7. Evaluation/ Reasoning
8. Identify a parallel argument/Structure.

1. IDENTIFY THE INFERENCE / MUST BE TRUE QUESTION

These type of questions are extremely common. An Inference means the same thing as “must be true”. Conclusions differ from inferences in that conclusions are the result of premises and inferences are something that must be true. The following are the typical Inference (Must be true) based Questions:

If the statements above are true, which of the following must also be true?

Which of the following is [implied, must be true, implicit, most reasonably drawn] in the passage above?

Which of the following inferences is best supported by the statement made above?

How to tackle “Identify the inference / Must be true questions”:

Read the stimulus and look for the argument.

Note that Must Be True questions may not contain an argument. They may just be a series of facts. Nevertheless, try to find the argument.

Avoid choices which contain absolute statements - never, always, none, only etc. Although these words might appear in some correct choice, you should be very sure about them.

Some of the options can be eliminated as they go beyond the scope of the passage. Note that an inference can be based on only some of the information provided and not the complete passage.

Stimulus Argument

Increases in funding for police patrols often lower the rate of crimes of opportunity such as petty theft and vandalism by providing visual deterrence in high-crime neighborhoods. Levels of funding for police patrols in some communities are increased when federal matching grants are made available.

Example 1

Which of the following can be correctly inferred from the statements above?

- (a) Areas with little vandalism can never benefit from visual deterrence.
- (b) Communities that do not increase their police patrols are at higher risk for crimes of opportunity late at night.
- (c) Federal matching grants for police patrols lower the rate of crimes of opportunity in some communities.
- (d) Only federal matching grants are necessary to reduce crime in most neighborhoods.

Solution : (c) (c) is a summary of the information provided; it is the logical end of a chain of reasoning started in the stimulus argument. The sequence of events goes like this : Increased funding → Increased visual deterrence → Lower crime

The last statement could be mapped as follows:

Federal grants → Increased patrol funds

(c) makes the chain complete by correctly stating that federal grants can lead to lower crime in some communities.

Now the logical chain becomes:

Federal grants → Increased funding → Increased visual deterrence → Lower crime

The other answer choices may not be correctly inferred because they go beyond the scope of the argument. They may be objectively, factually correct, or they may be statements that you would tend to agree with. However, you are limited to the argument presented when choosing a correct answer.

2. IDENTIFY THE ASSUMPTION.

An assumption is an unstated premise that supports the author's conclusion. It's the connection between the stated premises and the conclusion. An assumption is something that the author's conclusion depends upon. Assumption questions are extremely common and have types that look like this:

Which of the following most accurately states a hidden assumption that the author must make in order to advance the argument above?

Which of the following is an assumption that, if true, would support the conclusion in the passage above?

How to approach "Identify the assumption questions"

Look for gaps between the premises and the conclusion. Ask yourself why the conclusion is true. Before you progress to the answer choices, try to get feel of what assumption is necessary to fill that gap between the premises.

Beware of extreme language in the answer choices of assumption questions. Assumptions usually are not extreme. "Extreme" answer choices usually contain phrases such as always, never, or totally.

Stimulus Argument

Traditionally, decision making by doctors that is carefully, deductively reasoned has been considered preferable to intuitive decision making. However, a recent study found that senior surgeons used intuition significantly more than did most residents or mid-level doctors. This confirms the alternative view that intuition is actually more effective than careful, methodical reasoning.

Example 2 The conclusion above is based on which of the following assumptions?

- (a) Senior surgeons are more effective at decision making than are mid-level doctors.
- (b) Senior surgeons have the ability to use either intuitive reasoning or deductive, methodical reasoning in making decisions.
- (c) The decisions that are made by mid-level and entry-level doctors can be made as easily by using methodical reasoning as by using intuitive reasoning.
- (d) Senior surgeons use intuitive reasoning in making the majority of their decisions.

Solution: (a) The correct answer is (a), which provides a missing link in the author's reasoning by making a connection from the evidence: that intuition is used more by senior surgeons than other, less-experienced doctors, and the conclusion: that, therefore, intuition is more effective. None of the other choices helps bridge this gap in the chain of reasoning. Although some of the other statements may be true, they are not responsive to the question. In fact, they mostly focus on irrelevant factors such as appropriateness, ease of application, ability, etc.

3. STRENGTHEN ARGUMENT.

Assumptions connect premises to conclusions. An argument is strengthened by strengthening the assumptions. Here are some examples of Strengthen question types:

The conclusion would be more properly drawn if it were made clear that...

Which of the following, if true, would most strengthen the conclusion drawn in the passage above?

How to approach "Strengthen an argument"

Once you have identified the argument of the passage, i.e. the evidence(s) + conclusion, try putting in each option

with the argument. Check if the assumption(s) you have drawn is (are) strengthened if you accept the content of the option as true.

Stimulus Argument

Three years after the Bhakra Nangal Dam was built, none of the six fish species native to the area was still reproducing adequately in the river below the dam. Because the dam reduced the average temperature range of the water from approximately 40° to approximately 10°, biologists have hypothesized that sharp increases in water temperature must be involved in signaling the affected species to begin their reproduction activities.

Example 3 Which of the following statements, if true, would most strengthen the scientists' hypothesis?

- (a) The native fish species were still able to reproduce in nearby streams where the annual temperature range remains approximately 40°.
- (b) Before the dam was built, the river annually overflowed its banks, creating temporary backwaters that were used as breeding areas for the local fish population.
- (c) The lowest temperature ever recorded in the river prior to dam construction was 30°; whereas the lowest recorded river temperature after construction was completed has been 40°.
- (d) Non-native fish species, introduced after the dam was completed, have begun competing with the native species for food.

Solution: (a) (a) most strengthens the conclusion that the scientists reached. It does so by showing that there is a control group. In other words, a similar population, not subjected to the same change as the population near the dam, did not experience the same type of result. Here the basic assumption about the conclusion that scientists reached is that 'because of the reduction of average temperature range of the water, the reproduction of the native fish species has reduced drastically'. Option (a) clearly strengthens the assumption.

4. WEAKEN ARGUMENT.

Assumptions connect premises to conclusions. An argument is weakened by weakening the assumptions. Here are some examples of Weaken question types:

Which of the following, if true, would weaken the conclusion drawn in the passage above?

The argument as it is presented in the passage above would be most strengthened if which of the following were true?

How to approach "Weaken an argument"

Once you have identified the argument of the passage, i.e. the evidence(s) + conclusion, try putting in each option with the argument. Check if the assumption(s) you have drawn is (are) weakened if you accept the content of the option as true.

Stimulus Argument

A drug that is very effective in treating some forms of cancer can, at present, be obtained only from the bark of the Raynahu, a tree that is quite rare in the wild. It takes the bark of approximately 5,000 trees to make one pound of the drug. It follows, then, that continued production of the drug must inevitably lead to the Raynahu's extinction.

Example 4 Which of the following, if true, most seriously weakens the above conclusion?

- (a) The drug made from Raynahu bark is dispensed to doctors from a central authority.
- (b) The drug made from the Raynahu bark is expensive to produce.
- (c) The Raynahu generally grows in largely inaccessible places.
- (d) The Raynahu can be propagated from cuttings and cultivated by farmers.

Solution : (d) (d) provides an alternate source of the Raynahu bark. Even though the tree is rare in the wild, the argument is silent on the availability of cultivated trees. The author of the argument must be assuming that there are no Raynahu trees other than those in the wild, in order to make the leap from the stated evidence to the conclusion that the Raynahu is headed for extinction. The option (d) weakens the assumption - 'there are limited Raynahu trees' - by saying that there are other ways as well for the propagation of Raynahu. The other answer choices all contain information that is irrelevant. Note that the correct choice does not make the conclusion of the argument impossible. In fact, it is possible that there may be domesticated Raynahu trees and the species could still become extinct. Answer choice (d) is correct because it makes the conclusion about extinction less likely to be true.

5. CONCLUSION/MAIN POINT QUESTION

In Main Point / Conclusion questions, you have to identify the conclusion of an argument. You are trying to find the author's point and should approach this question in a similar way to the reading comprehension main point questions. They come in several different formats:

The main point of the passage is that...

Which of the following statements about... is best supported by the statements above?

Which of the following best states the author's conclusion in the passage above?

Which of the following conclusions can be most properly drawn from the data above?

The conclusion of arguments in Main Point questions is usually not directly stated. To find the conclusion, identify the premises and then identify the conclusion drawn from the premises. Main Point questions differ from the other Critical Reasoning questions in that the argument in the stimulus is usually valid. (In most other Critical Reasoning questions the reasoning is flawed.) Conclusion questions require you to choose the answer that is a summary of the argument.

How to approach "Main Point Questions":

Main Point answers must be within the scope of the passage.

Your opinions or information outside of the passage are always outside of the scope.

Some of the options given can be out of the scope of the passage.

Knock out answers with extreme wording. Main Point answers typically do not use *only*, *always*, *never*, *best* or any strong words that leave little room.

6. IDENTIFY THE PARADOX

These questions present you with a paradox, a seeming contradiction or discrepancy in the argument, and ask you to resolve it or explain how that contradiction could exist. In other words, there are two facts that are both true, and yet they appear to be in direct conflict with one another. Here are some examples of the ways in which these questions are worded:

Which of the following, if true, would help to resolve the apparent paradox presented above?

Which of the following, if true, contributes most to an explanation of the apparent discrepancy described above?

How to approach "Identify the paradox questions"

Read the argument and find the apparent paradox, discrepancy, or contradiction.

State the apparent paradox, discrepancy, or contradiction in your own words.

Use process of elimination. The best answer will explain how both sides of the paradox, discrepancy, or contradiction can be true. Eliminate answers that are out of scope.

Stimulus Argument

Town Y is populated almost exclusively by retired people and has almost no families with small children. Yet Town Y is home to a thriving business specializing in the rental of furniture for infants and small children.

Example 5 Which of the following, if true, best reconciles the seeming discrepancy described above?

- (a) The business specializing in the rental of children's furniture buys its furniture from distributors outside of Town Y.
- (b) The few children who do reside in Town Y all know each other and often stay over night at each other's houses.
- (c) Many residents of Town Y who move frequently prefer to rent their furniture rather than buy it outright.
- (d) Many residents of Town Y must provide for the needs of visiting grandchildren several weeks a year.

Solution : (d) The correct answer (d), explains why a town of mostly retired residents might need to rent children's furniture. The other answer choices all contain irrelevant information. This further illustrates the fact that, on all question types, if you eliminate the irrelevant choices, the remaining choice will most likely be correct.

Comprehension and Data Interpretation



General Comprehension

How to Attempt Reading Comprehensions?

"Comprehension questions judge your ability to understand the passage, analyse the problem in a proper perspective as well as your ability to answer quickly, correctly and systematically". Comprehension measures your ability to read and understand the written passage.

Answer the questions based on reading the passage, it is important that they should be based on the information given in the passage. Your common sense, knowledge and presumption should not be taken into account while answering the questions. If you have some information from your own experience and knowledge, you should not use it to answer the question. Even if you think that there is some mistake, you must still answer the question on the basis of the information given in the passage. There are certain techniques that will help you in answering the questions based on the given passages.

Important Techniques

- Use your pencil :** To begin with, use your pencil as a pointer. Using the pencil to guide your eye along a line of text helps you to focus on the details in the reading; it holds your attention to the precise words in the passage. In a long test, attention may weaken. Fatigue may blunt your attention to details. But using your pencil as a pointer will help you to preserve your attention to details.
- Increase in reading speed :** Another benefit of using the pencil as a pointer is that it will probably speed up your reading. The steady flow of the pencil across the page with each line of text draws the eye along at a steady pace. Do not go faster than you can grasp the text, but do try to keep your reading going at a steady pace set by the pencil.
- Circle key words and phrases :** Remember, you are not reading for just a vague general understanding of the passage. You usually have to read for detailed understanding. There will be individual words which are important for grasping a point exactly. Circle key words or phrases which will enable you to zero in on precise points needed to answer a question.
- Keep forging ahead :** Do not get bogged down if there is a word or sentence you do not understand. You may get the main idea without knowing the individual word or sentence. Sometimes you can sense the meaning of the word from the context. Sometimes the word or sentence may not be the basis of any question. If there is some idea you need to answer a question but do not understand, read it one more time. If you still do not understand it, move on.

You can come back to this question later if you have more time at the end of the test.

- Formation of an idea:** Another good reading comprehension strategy is to read the questions before starting the passage. This does not mean to read the answer choices at this time. By reading the questions, you will have an idea of what information you will need after reading the passage. This may alert you to certain details, ideas and specific areas in the paragraph where the questions are drawn from.

Right Approach to Answer the Questions

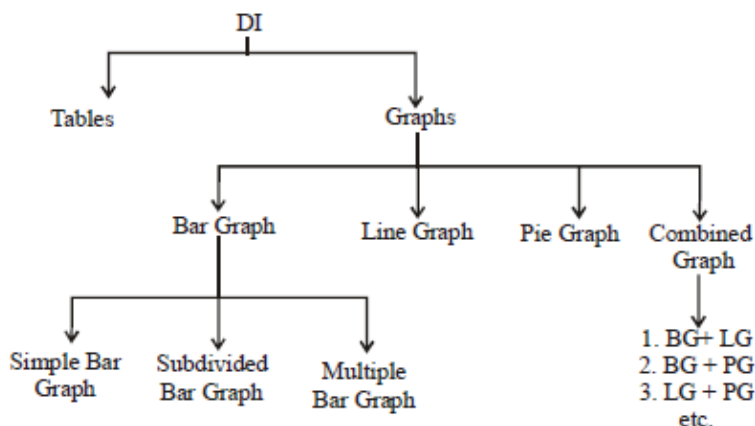
- Questions are to be answered on the basis of the information provided in the passage and you are not expected to rely on outside knowledge of a particular topic. Your own views or opinions may sometimes conflict with the views expressed or the information provided in the passage. Be sure that you work within the context of the passage. You should not expect to agree with everything you encounter in reading passages.
- You should analyse each passage carefully before answering the accompanying questions. As with any kind of close and thoughtful reading, look for clues that will help you understand less explicit aspects of the passage. Try to separate main ideas from supporting ideas or evidences.
- Note transitions from one idea to the next, and examine the relationships among the different ideas or parts of the passage. For example, are they contrasting? Are they complementary? Consider the points the author made, the conclusions drawn, and how and why those points are made or conclusions are drawn.
- Read each question carefully and be certain that you understand exactly what is being asked.
- Always read all the answer choices before selecting the best answer.
- The best answer is the one that most accurately and most completely answers the questions being posed. Be careful not to pick an answer choice simply because it is a true statement. Be careful also not to be misled by answer choices that are only partially true or only partially satisfy the problem posed in the question.
- The methodology of eliminatin wrong answers also works here. It simply means that if you are unable to judge the right choice or right answer, tick the answers that are incorrect, in this way if there remains one answer to be ticked, that will be the correct answer. But if there remains more than one answer to be ticked then it is up to you, whether you like to take risk or not. None can help you, as there may be negative marking also.



Data Interpretation

Data Interpretation questions are based on the information given in the tables and graphs.

Classification of Data Interpretation :



TABLES

A table is one of the easiest way for summarising data.

A statistical table is the logical listing of related quantitative data in vertical columns and horizontal rows of numbers with sufficient explanatory and qualifying words, phrases and statements in the form of titles, heading and notes to make clear the meaning of data.

REMEMBER

$$\text{Average} = \frac{\text{Sum of all items}}{\text{Total number of items}}$$

λ = The bars are drawn proportional in length to the total and then divided in the ratios of their components.

$$\% \text{ change (increase or decrease)} = \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100$$

GRAPHS

Graphs are a convenient way to represent information. The graphs should be labelled properly to show what part of the graphs shows what value.

1. **Bar Graphs** - Bar diagram consist of a number of equidistant rectangles. One for each category of the data in which the magnitudes are represented by the length or height of rectangle, whereas width of rectangles are immaterial. Thus, a bar is just one dimensional as only the length of the bar is to be considered and not the width. All the bars drawn in a diagram are generally of uniform width which depends on the number of bars to be constructed and the availability of the space.

Types of Bar Graphs are-

- (i) **Simple Bar Graph** : It is used to represent only one dependent variable.
- (ii) **Sub-divided Bar Graphs** : These are used to represent the break down of a total into its component bars. A bar is divided into different segments, each segment representing a given component of the total different shades, colours, designs etc. are used to distinguish the various components. An index is given to represent the various components. To compare, the order of various components in the different bars is same.

- (iii) **Multiple Bar Graph (MBG)** : When a combination of inter-related variables are to be represented graphically, multiple bar diagrams are used. These are extended form of simple bar diagrams. In M.B.G. many aspects of the data are presented simultaneously with separated bars of various shades of colours. An index is given to explain the shades & colours used.
2. **Line Graph(LG)** are more obvious, precise & accurate than the diagrams. LG are used to show how a quantity changes very often the quantity is measured as time changes. If the line goes up, the quantity is increasing. If the line is horizontal, the quantity is not changing.
3. **Pie Graph(PG)** is a pictorial representation of numerical data by non-intersecting adjacent sectors of circle area of each sector is proportional to the magnitude of the data represented by the sector.

$$1\% \text{ of total value} = \frac{360}{100} = 3.6^\circ$$

The % of components parts can be converted to degrees by multiplying each them by 3.6° .

$$\text{Degree of any component part} = \frac{\text{component value}}{\text{Total value}} \times 360.$$

Solved Example

Directions (Qs. for Examples 1 to 4) : These examples are based on the table given below.

The following table gives the sales details for text books and reference books at Primary / Secondary / Higher Secondary / Graduate levels

Year	Primary	Secondary	Higher Secondary	Graduate Level
2009	42137	8820	65303	25343
2010	53568	10285	71602	27930
2011	58770	16437	73667	28687
2012	56872	15475	71668	30057
2013	66213	17500	78697	33682
2014	68718	20177	82175	36697

Example 1 What is the growth rate of sales of books at primary school level from 2009 to 2014 ?

- (a) 29% (b) 51% (c) 63% (d) 163%

Solution :

$$(d) \text{ Growth rate} = \frac{68718}{42137} \times 100 = 163\%$$

Example 2 Which of the categories shows the lowest growth rate from 2009 to 2014 ?

- (a) Primary (b) Secondary (c) Higher Secondary (d) Graduate level

Solution :

$$(c) \text{ Growth \% (Primary)} = \frac{68718}{42137} \times 100 = 163\%$$

$$\text{Growth \% (Secondary)} = \frac{20177}{8820} \times 100 = 204\%$$

$$\text{Growth rate (Higher secondary)} = \frac{82175}{65303} \times 100 = 125\%$$

$$\text{Growth rate (Graduate level)} = \frac{36697}{25343} \times 100 = 144\%$$

Example 3 Which category had the highest growth rate in the period ?

- (a) Primary (b) Secondary (c) Higher Secondary (d) Graduate level

Solution :

(b)

Example 4 Which of the categories had either a consistent growth or a consistent decline in the period shown ?

- (a) Primary (b) Secondary (c) Higher Secondary (d) Graduate level

Solution :

(d)

Directions (for Examples 5 to 9) : These questions are based on the table & information given below :

The amount of money invested (in rupees crore) in the core infrastructure areas of two districts, Chittoor and Khammam, in Andhra Pradesh, is as follows :

Chittoor District			Khammam District		
Core Area	2012	2013	Core Area	2012	2013
Electricity	815.2	1054.2	Electricity	2065.8	2365.1
Chemical	389.5	476.7	Chemical	745.3	986.4
Thermal	632.4	565.9	Thermal	1232.7	1026.3
Solar	468.1	589.6	Solar	1363.5	1792.1
Nuclear	617.9	803.1	Nuclear	1674.3	2182.1
Total	2923.1	3489.5	Total	7081.6	8352

Example 5 By what percent was the total investment in the two districts more in 2013 as compared to that in 2012 ?

- (a) 14% (b) 21% (c) 24% (d) 18%

Solution :

(d) Total investment in 2012 = 2923.1 + 7081.6 = 10004.7

Total investment in 2013 = 3489.5 + 8352.0 = 11,841.5

$$\therefore \% \text{ increase} = \frac{11841.5 - 10,004.7}{10,004.7} \times 100 = 18.36\%$$

Example 6 Approximately how many times the total investment in Chittoor was the total investment in Khammam ?

- (a) 2.8 (b) 2.0 (c) 2.4 (d) 1.7

Solution :

(c) Total investment in Chittoor = 6412.6

Total investment in Khammam = 15433.6

$$\therefore \frac{\text{Total investment in Khammam}}{\text{Total investment in Chittoor}} = \frac{15433.6}{6412.6} = 2.40$$

Example 7 The investment in Electricity and Thermal Energy in 2012 in these two districts formed what percent of the total investment made in that year ?

- (a) 41% (b) 47% (c) 52% (d) 55%

Solution :

(b) Investment in electricity & thermal energy in 2012 in two districts

= 815.2 + 632.4 + 2065.8 + 1232.7 = 4746.1

$$\% \text{ in terms of total investment} = \frac{4746.1}{10,004.7} \times 100 = 47.43\%$$

